Using multiple attractor chaotic systems for communication

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Abstract

In recent work with symmetric chaotic systems, we synchronized two such systems with one-way driving. The drive system had 2 possible attractors, but the response system always synchronized with the drive system. In this work, we show how we may combine 2 attractor chaotic systems with a multiplexing technique first developed by Tsimring and Suschick to make a simple communications system. We note that our response system is never synchronized to our drive system (not even in a generalized sense), but we are still able to transmit information. We characterize the performance of the communications system when noise is added to the transmitted signal.

1. Introduction

It has been suggested recently that chaotic systems might be useful for communications [1-7]. There are many practical problems that arise when a chaotic signal is transmitted. Among these problems is additive noise. In conventional digital communications systems, one tries to decide which of several symbols has been transmitted in a noisy environment using the principal of maximum likelihood [8]. If there are several possible symbols that might have been transmitted, the most likely symbol is taken to be the received symbol. Naturally, this estimation is easier if the symbols are far apart in some symbol space. For our chaotic communications system, we use two widely separated attractors for our two symbols. We then combine signals from two chaotic systems so that our transmitted signal has no DC component. We follow this procedure with two different chaotic systems, and compare signal to noise performance.

2. Multiple attractor systems

The basic principle that we will use has been described previously [9] in a 4-dimensional circuit. The circuit had a symmetric nonlinearity, so that for some parameters the circuit had two symmetric attractors. We built a drive circuit which drove a response circuit through a one-way driving. Normally, one would expect that the response circuit would also have two attractors, so that the response would not synchronize to the drive unless the response circuit was in the correct basin of attraction. For some parameters, however, the out-of-sync attractor in the response circuit was near neutral stability. After a few cycles in the out-of-sync attractor, the response system converged to the in sync attractor.

.In order to have a drive signal with no DC component, we use a technique of Tsimring and Suschick [6] to add signals from two chaotic systems in order to cancel the DC components.

3. General layout

Figure 1 is a block diagram of our technique applied to a pair of 3-dimensional chaotic systems. Drive systems A and B do not have to be identical, although in this paper we will use identical systems for simplicity. A and B are both symmetric nonlinear systems (they do not have to be chaotic) with 2 attractors each. We form a linear combination of signals from A and B. We choose the linear combination so that the DC level of the transmitted signal u is 0.

If we have identical systems in opposite attractors, this requires that $k_4 = k_1$, $k_5 = k_2$, and $k_6 = k_3$. For non identical systems, the *k's* must be chosen appropriately so that the time average of *u* is 0. The idea of making a linear combination of drive variables comes from control theory [10] and the work of Peng et al. [11] who used this technique to synchronize hyperchaotic systems. We have shown [12] that such a technique can make the response system very stable and insensitive to parameter mismatch.

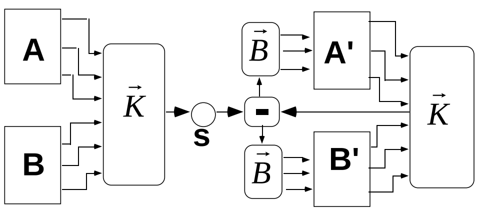

Fig. 1 Block diagram of a 2-attractor chaotic communications system. A and B are chaotic systems in the transmitter, while A' and B' are chaotic systems in the response. The information signal, *s*, is ± 1.

To change symbols, we can either flip the attractors in A and B or we can invert the transmitted signal *u*, which is equivalent. We multiply the transmitted signal *u* by *s* = ±1 to produce $u_S = su$. The signal *s* is our binary information signal.

The response systems are A' and B'. We make an identical linear combination of variables from A' and B' to make u' , and generate a difference signal $v = u_S - u'$. The difference signal is multiplied by one of the constants b_i (i = 1,6) and fed back into the response systems. The *k's* and *b's* are chosen according to techniques in [13].

Confirming an observation of Tsimring and Suschick [6], we show in [13] that multiplexing by adding chaotic signals always results in an unstable response system when the response system consists of two identical chaotic systems. Because of the structure of the coupling, the eigenvalues of one of the jacobians of the chaotic systems will also be the eigenvalue of the chaotic response system, so the response system can never be stable (for identical systems).

Because the response systems are not stable, our response systems A' and B' actually do not synchronize to the drive systems A and B. In our case, this lack of synchronization is not a problem because we are not interested in synchronization itself but rather in determining which attractors the drive systems are in. We will see below that even without synchronization, we have more than enough information to determine the drive system attractors. We show in [13] how to couple the drive and response systems so that the response system motion remains bounded. We note that we could also use periodic attractors for our communications system, which might change the instability problem.

4. Symmetric Rossler system

Two Attractor Signaling

We first use a 3-dimensional system that is similar to the Rossler system. Our symmetric Rossler system has a symmetric piecewise nonlinearity. The system is described by (i = 0,1, $j = 3i + 3$):

$$\begin{aligned} \frac{dx_j}{dt} &= -\alpha_i\left[0.05x_j + 0.5x_j + x_j\right] \\ \frac{dx_j}{dt} &= -\alpha_i\left[-x_j - \rho x_j\right] \\ \frac{dx_j}{dt} &= -\alpha_i\left[x_j - g_1\left(x_j\right)\right] \\ u &= \sum_{j=1}^{6} k_j x_j \quad u_s = su \qquad s = \pm 1 \end{aligned} \qquad (1)$$

where $g_1(x)$ is $5(x + 3)$ if x > -3, $5(x - 3)$ if $x > 3$, and 0 otherwise, $\rho = 0.25$, $\alpha_0 = 1.2$ and $\alpha_1 = 1$. The *k* parameters are given below. There are two chaotic systems (i = 0 and i = 1) corresponding to A and B in Fig. 1. Each drive system was in an opposite attractor.

We numerically integrated eqs. (1) with a 4-th order Runge-Kutta integration routine [14] with a time step of 0.2 s. Figure 2(a) shows the one of the attractors from the symmetric Rossler system, while Fig 2(b) shows the other. We will call these two attractors the + attractor and the - attractor.

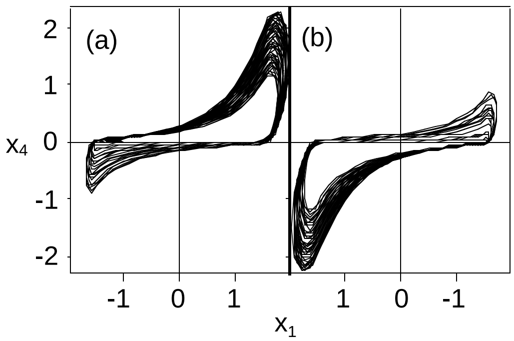

Fig. 2 (a) is the + attractor for the symmetric Rossler system of eq. (1). (b) is the - attractor for the symmetric Rossler system of eq. (1).

The response system is described by

$$
\begin{aligned}
&v = u_s - u' \\
&\frac{dx'_j}{dt} = -\alpha_i\left[0.05x'_j + 0.5x'_j + x'_j - b_j v\right] \\
&\frac{dx'_j}{dt} = -\alpha_i\left[-x'_j - \rho x'_j - b_j v\right] \qquad (2) \\
&\frac{dx'_j}{dt} = -\alpha_i\left[x'_j - g_1\left(x'_j\right) - b_j v\right] \\
&u' = \sum_{j=1}^{6} k_j x'_j
\end{aligned}
$$

where the parameters are the same as in eq. (1). The parameter $s = \pm 1$ is our binary information signal, as described above. The k's were $k_1 = -1.2824$, $k_2 = 1.91712$, $k_3 = 1.19166$, $k_4 = k_1$, $k_5 = k_2$, and $k_6 = k_3$ and the b's were $b_1 = 1.09793$, $b_2 = 0.65328$, $b_3 = 0$, $b_4 = 1.62025$, $b_5 = 1.12384$, $b_6 = 0$.

To decode the transmitted message, we simply track whether system A' is in the + attractor or the - attractor. To aid in the detection, we use a low pass filter. We can change the value of s at time $t = nT$, where T is one clock period. Our detector is described by

$$
\begin{aligned}
&\frac{dw}{dt} = x'_3 - w \\
&n = 0,1,2,\ldots \qquad (3) \\
&\textbf{if } t = nT \textbf{ then } w = 0
\end{aligned}
$$

At time nT , just before resetting, a positive value of w indicates that $s = +1$, while a negative value of w indicates $s = -1$.

We characterized the performance of our communications system when subject to noise by calculating the probability of bit error P_b as a function of the ratio of bit energy E_b to noise power spectral density N_0 . We integrated eqs. (1-3) for 800,000 steps with a time step of 0.2 s. We set the value of s at +1 and reset the detector variable w to 0 every T = 40 s. We measured the value of w just before resetting. If the value of w was not greater than 0, a bit error was recorded.

We added Gaussian noise to the transmitted signal u . We changed the variance of the noise to change the power spectral density N_0 . We calculated the bit energy P_b by finding the average power in the transmitted signal and multiplying by the data period T. In Figure 3 we plot the bit error rate P_b as a function of E_b/N_0 . For comparison, we also plot P_b for a bipolar binary baseband signaling system, as calculated in [8]. A "bipolar binary baseband" signal consists of sending +1 or -1, as if we were transmitting only s.

Parameter Modulation Signaling

As an additional comparison in Fig. 3, we used the system of eqs. (1-3) to transmit information using parameter modulation. We switched the parameter ρ in the transmitter between 0.25 and 0.2, while keeping all parameters in the receiver fixed. When ρ was 0.2, the transmitter and receiver were not matched, and so did not synchronize. We used the error signal v in eq. (2) to detect the information signal. Our detector was again a low pass filter that used the square of v . We added Gaussian white noise to the transmitted signal as before and ran numerical simulations to find the bit error probability P_b as a function of E_b/N_0 . We plot these results in Fig. 3.

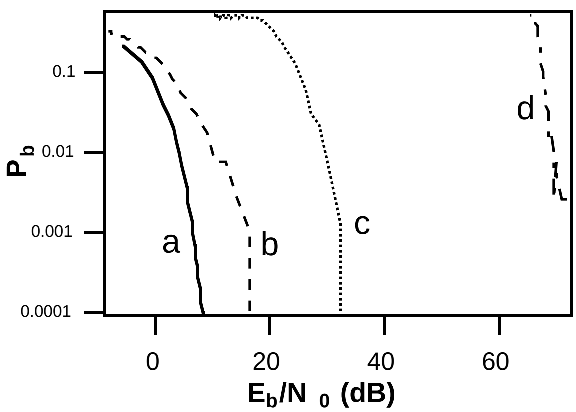

Fig. 3 Probability of bit error P_b as a function of energy per bit E_b divided by noise power spectral density N_0 for several different communications systems. (a), an analytic example for a bipolar baseband signal from [8], shown for comparison.(b), the result for ta he 4-D system two attractor switching. (c), the result for the symmetric Rossler system of eqs (1-3) using two attractor signaling. (d), the result for the symmetric Rossler system of eqs. (1-3) using parameter modulation.

5. Alternate Circuit.

We also used a chaotic system described in [9]. The + and - attractors in the chaotic system of ref [9] are better separated than the attractors for the piecewise linear Rossler system. We calculate P_b as a function of E_b/N_0 for this new system. As can be seen on Fig. 3, the system with better seperated attractors does perform better.

8. Conclusions

We have shown that using attractors for symbols can improve the noise robustness of a chaotic communications system by several orders of magnitude.

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