Communicating using filtered synchronized chaotic signals.

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Abstract- The principles of synchronization of chaotic systems are extended to the case where the drive signal is filtered. A feedback loop in the response system with an identical filter is used to reconstruct the original drive signal, allowing synchronization. A simple parameter switching scheme is used to send information from a drive circuit to a receiver. It is also possible to add a chaotic signal with very similar frequency characteristics and still detect information encoded in the original chaotic carrier (but not the added chaotic signal), demonstrating the possibility of adding and separating multiple chaotic carriers with similar frequency characteristics.

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I. Introduction

Two chaotic circuits may be synchronized by driving a subsystem of a chaotic circuit with a signal from the full circuit. [1-7]. The chaotic subsystems may be cascaded so that the driving signal is reproduced, giving a test for whether or not the chaotic systems are synchronized [4, 8]. Others have shown how this work may be applied in simple communications systems [9-12] or non autonomous (periodically forced) chaotic systems [13]. One advantage of synchronizing non autonomous chaotic systems is that the periodic forcing terms for the two chaotic systems may be synchronized even when a large amount of noise or chaos is added to the driving signal. Synchronization may be even be achieved when the driving signal has been altered by a filter [14], as long as the original driving signal is reconstructed at the receiver. In this work, this filtered synchronization technique is used to separate a chaotic signal carrying information from a contaminating signal. It is shown that a receiver may be built that is only sensitive to the information carrier, rejecting the contaminating signal, even when it is another information-carrying chaotic signal. This raises the possibility of using multiple chaotic signals as broad-band carrier signals which occupy the same part of frequency space.

II. Theory of Synchronization

The theory of the synchronization of chaotic systems is described in detail elsewhere [2], so only a brief description is included here. We begin with a dynamical system that may be described by the ordinary differential equation

$$\dot{u}(t) = f(u) \qquad (1)$$

The system is then divided into two subsystems, u= (v,w);

$$\begin{aligned} \dot{v} &= g(v,w) \\ \dot{w} &= h(v,w) \end{aligned} \qquad (2)$$

where $v=(u_1,...,u_m)$, $g=(f_1(u),...,f_m(u))$, $w=(u_{m+1},...,u_n)$, and $h=(f_{m+1}(u),...,f_n(u))$. The division is truly arbitrary since the reordering of the u_i variables before assigning them to v, w, g, and h is allowed.

A first response system may be created by duplicating a new sub-system w’ identical to the w system, substituting the set of variables v for the corresponding v’ in the function h, and augmenting Eqs. (2) with this new system, giving,

$$\begin{aligned} \dot{v} &= g(v,w) \\ \dot{w} &= h(v,w) \\ \dot{w}' &= h(v,w'). \end{aligned} \tag{3}$$

If all the Lyapunov exponents of the w' system (as it is driven) are less than zero, then $w' - w \to 0$ as $t \to \infty$.

It is possible to take this system further. One may also reproduce the v subsystem and drive it with the w' variable [4], giving

$$\begin{aligned} \dot{v} &= g(v,w) \\ \dot{w} &= h(v,w) \\ \dot{w}' &= h(v,w') \\ \dot{v}'' &= g\left(v'', w'\right) \end{aligned} \tag{4}$$

If all the Lyapunov exponents of the w', v'' subsystem are less than 0, then $v'' \to v$ as $t \to \infty$. The example of eq. (4) is referred to as cascaded synchronization [8].

III. Filtering and synchronization

It is possible for chaotic circuits to produce signals which have large components at certain frequencies or other distinct spectral properties. This is especially true for non autonomous chaotic circuits [13]. The periodic peaks in the spectrum of the chaotic signal may be removed by a filter, but this alters the chaotic signal enough so that synchronization in the receiver does not occur. The periodic peaks also are a feature which could be used to detect the chaotic signal, which may not be desirable. Finally, if one could filter several chaotic signals to remove features that they had in common, it might be possible to add them together and later separate them based on their differences.

A basic scheme by which a filtered chaotic signal could be reconstructed was demonstrated in [14]. In this work, a set of band stop filters were used to remove the forcing frequency and the first four harmonics from a chaotic driving signal produced by a non autonomous chaotic circuit. The general filtering arrangement is shown in Fig. 1; to effect a band stop filter, the chaotic driving signal is first sent through a bandpass filter. The signal within this pass band is then subtracted from the chaotic driving signal and transmitted. To reconstruct the driving signal, the output of the response circuit is sent through a filter identical to the first bandpass filter. The filter output is added to the transmitted signal to produce a reconstructed driving signal, which is used to drive the response circuit. The response circuit will synchronize to the drive circuit if the feedback arrangement for the response circuit is stable in the synchronized state. These general concepts are not limited to band stop filters, although each combination of chaotic circuit and filter must be checked for stability.

The circuit used in [14] may be used to demonstrate a simple communications system using filtered synchronization. The basic chaotic circuit from [13] is known as the augmented Duffing (ADF) circuit. The ADF circuit is used with the band stop filters shown in Fig. 2, as well as an intermediate circuit to add in noise or another chaotic signal as a contaminating signal. The signal x first passes through a 2nd order band pass filter [15] and the filter output is then subtracted from the x signal to produce a driving signal x_t with a particular band of frequencies suppressed. Five bandpass filters were used, one at the driving frequency of 780 Hz and one at each of the first 4 harmonics. The forcing signal was subtracted from the x signal before filtering to further attenuate the component of the x_t signal at the forcing frequency. The equations for the drive, response and band pass filter for the ADF circuit were

$$\frac{dx}{dt} = \beta(y - z) \tag{5}$$

$$\frac{dy}{dt} = \beta\left(-\Gamma_y y - g(x) + \alpha\cos\left(\omega t + \phi\right) + A\right) \tag{6}$$

$$\frac{dz}{dt} = \beta\left(f(x) - \Gamma_z z\right) \tag{7}$$

$$w = \frac{d(x - \alpha\cos(\omega t))}{dt} \tag{8}$$

$$\frac{du_i}{dt} = \frac{-2.0}{R_{i2}C}u_i - \frac{1}{R_{i2}C}\left(\frac{1}{R_{i3}C} + \frac{1}{R_{i1}C}\right)v_i - \frac{1}{R_{i1}C}w \tag{9}$$

$$\frac{dv_i}{dt} = u_i \tag{10}$$

$$x_t = x + \sum_{i=1}^{5} v_i \tag{11}$$

$$x_d = x_t - \sum_{i=1}^{5} r_i \tag{12}$$

$$\frac{dq_i}{dt} = \frac{-2.0}{R_{i2}C}q_i - \frac{1}{R_{i2}C}\left(\frac{1}{R_{i3}C} + \frac{1}{R_{i1}C}\right)r_i - \frac{1}{R_{i1}C}\frac{dx''}{dt} \tag{13}$$

$$\frac{dr_i}{dt} = q_i \tag{14}$$

$$\frac{dz'}{dt} = \beta\left(f(x_d) - \Gamma_z z'\right) \tag{15}$$

$$\frac{dx''}{dt} = \beta\left(y'' - z'\right) \tag{16}$$

$$\frac{dy''}{dt} = \beta\left(-\Gamma_y y'' - g(x'') + \alpha\cos\left(\omega_r t + \phi_r\right) + A\right) \tag{17}$$

Equations (5-7) represent the chaotic driving circuit, eqs. (8-11) represent the drive system filters, eqs. (12-14) represent the response system filters and eqs. (15-17) represent the response circuit. The reconstructed driving signal is x_d. The variables R_{ji} are defined for each of the bandpass filters in table I. The actual resistor values were tuned with 20 turn potentiometers to adjust for errors in the capacitors. The value of C was 0.01 μF, and A was initially 0.0 V. The Q factor for each band pass filter was 20.

Figure 3 shows the power spectrum of the transmitted signal from the ADF circuit, x_t. The periodic component at the forcing frequency has been attenuated by about 35 dB. Without filters present, the phase ϕ_r of the response forcing must match the phase ϕ of the drive circuit forcing for synchronization to take place. A simple controller for matching phases in non autonomous chaotic circuits was demonstrated in [13]. Numerically, this controller is described by:

$$\Delta = \frac{1}{T_c}\int h\left(x_t, x'', t\right)dt \qquad (18)$$

$$h\left(x_t, x'', t\right) = x_t(\tau_n) \text{ for } \tau_n \le t < \tau_{n+1} \qquad (19)$$

where τ_n is the n'th time at which x'' crosses zero in the negative direction and T_c, the controller time constant, is 1 s. The function $h(x_t, x'', t)$ is a step wave function consisting of the value of x_t at the last time that x'' crossed zero. The signal Δ is the average of this step wave function. If x_t is unrelated to x'', then the average signal Δ will be zero, and no correction will take place. The error signal Δ is applied to the frequency modulation input of the function generator providing the response forcing signal.

IV. Improving synchronization

The setup of eqs. (5-17) is sufficient for demonstrating the principles of synchronization, but its performance was not good enough for communications. As a guide to the likely synchronization quality, the Lyapunov exponents for the filtered system were calculated from eqs. (5-17). In order to keep the number of variables manageable, only the filter at the fundamental forcing frequency of 780 Hz was used in the Lyapunov exponent calculations. The largest Lyapunov exponent for the response system was found to be -10 s^{-1}, which is much larger than the largest exponent for the unfiltered response system of -780 s^{-1}. With a global exponent so close to zero, the response circuit was very sensitive to local instabilities. Local instabilities may not have a large effect on synchronization unless there is some other nearby attractor that the chaotic system might be attracted to. The attractor for the ADF circuit did have a two-lobed structure (Fig. 4). The circuit moved between lobes infrequently (several forcing cycles usually passed between crossings), so if a local instability put the drive and response circuits in different lobes of this attractor, they might stay apart for a long time. The two possible solutions to this problem considered here were reducing the largest response Lyapunov exponent or reducing the symmetry of the attractor so that it was more one-sided, causing the local instabilities to have less effect.

One way to change the largest response Lyapunov exponent is to change the Q factor of the filter. Increasing the damping of the filter by decreasing the Q factor to 2.0 (R_1 = 20,300; R_2 = 41,900; R_3 = 10,260) was the first simulated modification attempted. This modification increased the largest Lyapunov exponent of the response system to190 s^{-1}, making it unstable. Increasing the Q factor of the filter to 200 dropped the largest exponent slightly to -16 s^{-1}. These results make sense when one considers that higher Q filters have a smaller effect on the dynamical system.

Lowering the symmetry of the dynamical system appeared to be an easier way to improve synchronization. The offset term A in eq. (6) and (17) was set to 1.0 V, resulting in improved synchronization by making the attractor more one sided. Figure 5 shows the attractor when the offset was added to the drive. When this offset was simulated in order to calculate the response Lyapunov exponents, it appeared to make the response system go unstable. It is not known why the simulation gave different results than the experiment, although the full filter was not used in the simulation.

Figure 6 shows the improvement in synchronization when A was set to 1.0 V in the circuit. Figure 6(a) shows x'' vs. x from the circuit for A=0.0 V, while 6(b) shows x'' vs. x when A=1.0 V.

V. Signal separation and communications

In previous work [13, 14], this controller was used only to lock onto a chaotic signal, in some cases when noise was present. In this work, this controller is used to distinguish between signals from different chaotic circuits using a property mentioned above: if the transmitted signal x_t is completely unrelated to the output signal x'', then the error signal Δ will be zero, and no correction will take place. For transmitted signals x_t that are not completely different from x'', different results are possible. If, for example, the chaotic driving circuit is changed only slightly, then there is still a close enough relation between x_t and x'' that Δ will not be zero. Because of this, Δ may be used to track small parameter changes in the driving chaotic circuit. If a contaminating signal from another chaotic circuit is added to x_t, then it is still possible that Δ will respond to x_t as long as the contaminating signal is not too large and the chaotic system that produces the contaminating signal is not too similar to the driving chaotic circuit. Although nonlinear systems are not actually orthogonal, this idea is similar in spirit to orthogonality. The filtering process helps accentuate the differences between chaotic systems by allowing the common features to be removed.

Several authors have demonstrated digital communication between cascaded chaotic circuits via parameter switching in the sending circuit [16-18]. Parameter switching may also be used with the filtered non autonomous chaotic circuits. The most likely parameter to be switched would seem to be the phase of the periodic forcing in the driving system, but the control system of [13] will lock in a stable fashion when the response forcing is in phase or 180 degrees out of phase with the drive, so phase switching was not used. The forcing offset A in eq. (6) and (17) was switched between $\pm$ 1.0 V, and the parameter switching was detected by monitoring the error signal Δ. Figure

7 shows the offset signal A as a time series and the resulting error signal coming from the response system controller. The switching speed is limited by the time constant in eq. (18), about 1 s for this system.

To demonstrate signal separation as described above, a second Duffing circuit with different parameters was built. This second Duffing circuit (the Single Well Duffing circuit, or SWD circuit) was described by the equations:

$$\frac{d\xi}{dt} = 10^4\psi \qquad (20)$$

$$\frac{d\psi}{dt} = 10^4\left(\beta\cos\left(\omega t + \phi_2\right) + A_2 - 0.256\psi - \xi^3\right) \qquad (21)$$

where β was 6.20 V and A_2 was 0.5 V. This second Duffing circuit was forced with an independent periodic forcing source at 780 Hz, so the phase ϕ_2 was not the same as ϕ in eq. (6). The ξ signal was filtered with a band stop filter to remove the forcing frequency and the first 4 harmonics to produce ξ_t , which was then added to the transmitted signal x_t with the same amplitude as x_t. Figure 8 is the power spectrum of the ξ_t signal from the SWD circuit. Synchronization of the periodic forcing in the drive and response systems was not lost when the ξ_t signal was added to the x_t signal with the same amplitude. Synchronization was lost for larger amplitudes of the ξ_t signal, but synchronization did not occur when the ξ_t signal alone from the SWD circuit was used to drive the response ADF circuit..

Figure 9 shows the offset signal A and the error signal Δ when the drive signal is the sum of x_t and ξ_t. The parameter switching is still detectable in the error signal Δ. In order to test the ability of this system to separate chaotic communications signals, the offset parameter A in eq. (6) (the ADF circuit) was held constant at 1.0 V while the offset parameter A_2 in eq. (21) (the SWD circuit) was switched between $\pm$ 0.5 V. If all other controller parameters were left the same (so that the ADF response was still synchronized to the ADF drive), this switching signal could not be detected in the error signal Δ.

If the frequency of the response periodic forcing was set to about 760 Hz, so that the drive and response were no longer synchronized, the switching signal when the offset A_2 was switched could be detected in the error signal Δ. The response was not synchronized to either circuit in this case, but the ξ_t signal from the SWD circuit was related to the output x" of the ADF response circuit, so it contributed a finite component to the error signal Δ. The ADF and SWD circuits were similar enough that the signal from the wrong one could be detected in some cases. This does raise the question of how different the chaotic systems must be to make signal separation possible. Some tolerance to mismatch between drive and response circuits is necessary, but this tolerance also allows the detection of signals from similar circuits. Improving the match between drive and response circuits would allow the reduction of the mismatch tolerance, improving the selectivity of the response circuit.

VII. Conclusions

The band stop filtering described here is only one example of a more general class of transformations that may be applied to chaotic signals used for synchronization. Other examples are given in [14]. Provided the response system remains stable, chaotic signals used for synchronization may be transformed in a variety of ways, such as low, high or band pass filtering, that may make them more useful for communications.

Signal separation was also demonstrated in this work. Filtering chaotic signals allows the removal of spectral features that make signal separation difficult. In the band stop filtering example, removing spectral features that both chaotic signals had in common allowed their differences to be used in signal separation. This work is still at a very preliminary stage; issues that must still be investigated include just how different the chaotic signals must be to be separable, how much noise can be present, the effects of non additive noise (such as multipath or phase noise), how to design the optimum filters and response circuits, and many other issues. In this simple study, filtered synchronized chaotic signals do show some promise for broad band communications.

It has been noted [19] that signal masking using chaotic signals may be easy to defeat. Short studied several types of nonchaotic signals buried in chaos and found that he could reproduce the original chaotic signal and therefore recover the nonchaotic signal. Making the nonchaotic signal smaller only made this easier, as it made reproducing the chaotic signal easier. The one case where he could not separate the signals was when both signals were chaotic. This suggests that signal masking may be more effective when a sum of chaotic signals is used, as described in this work. In most other chaotic communications methods, noise resistance, jamming resistance and unmasking resistance have not been demonstrated.

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Figure Captions

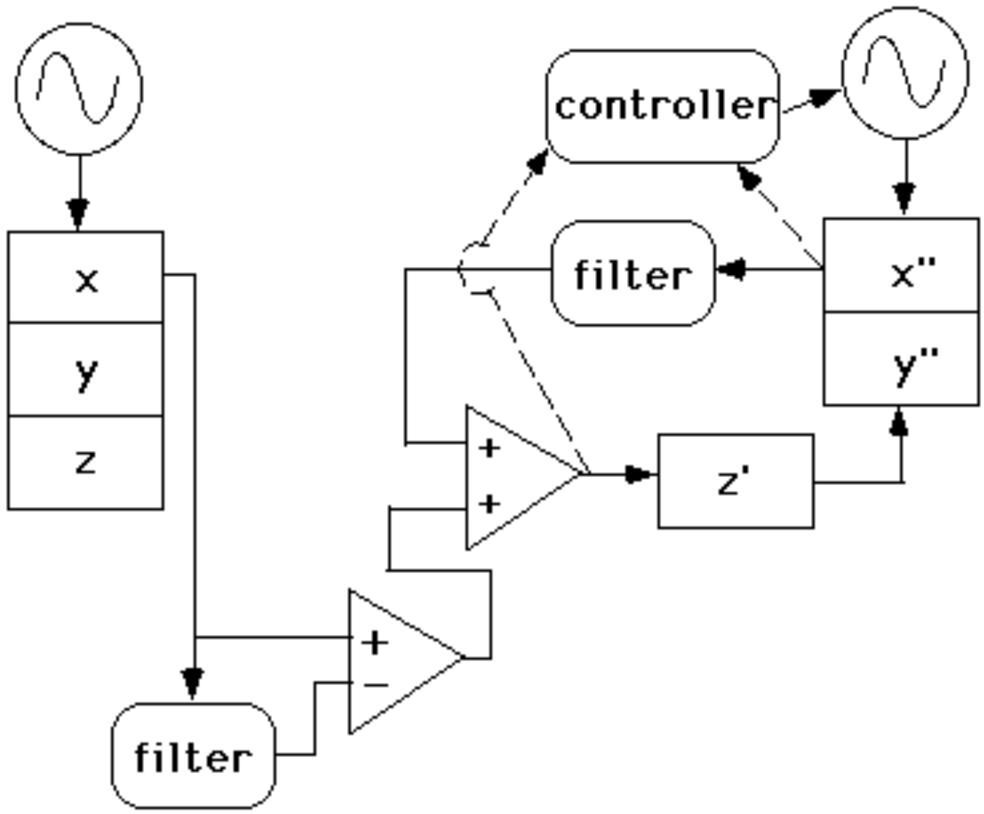


Fig. 1. Block diagram of the filtering arrangement used with the ADF circuit.

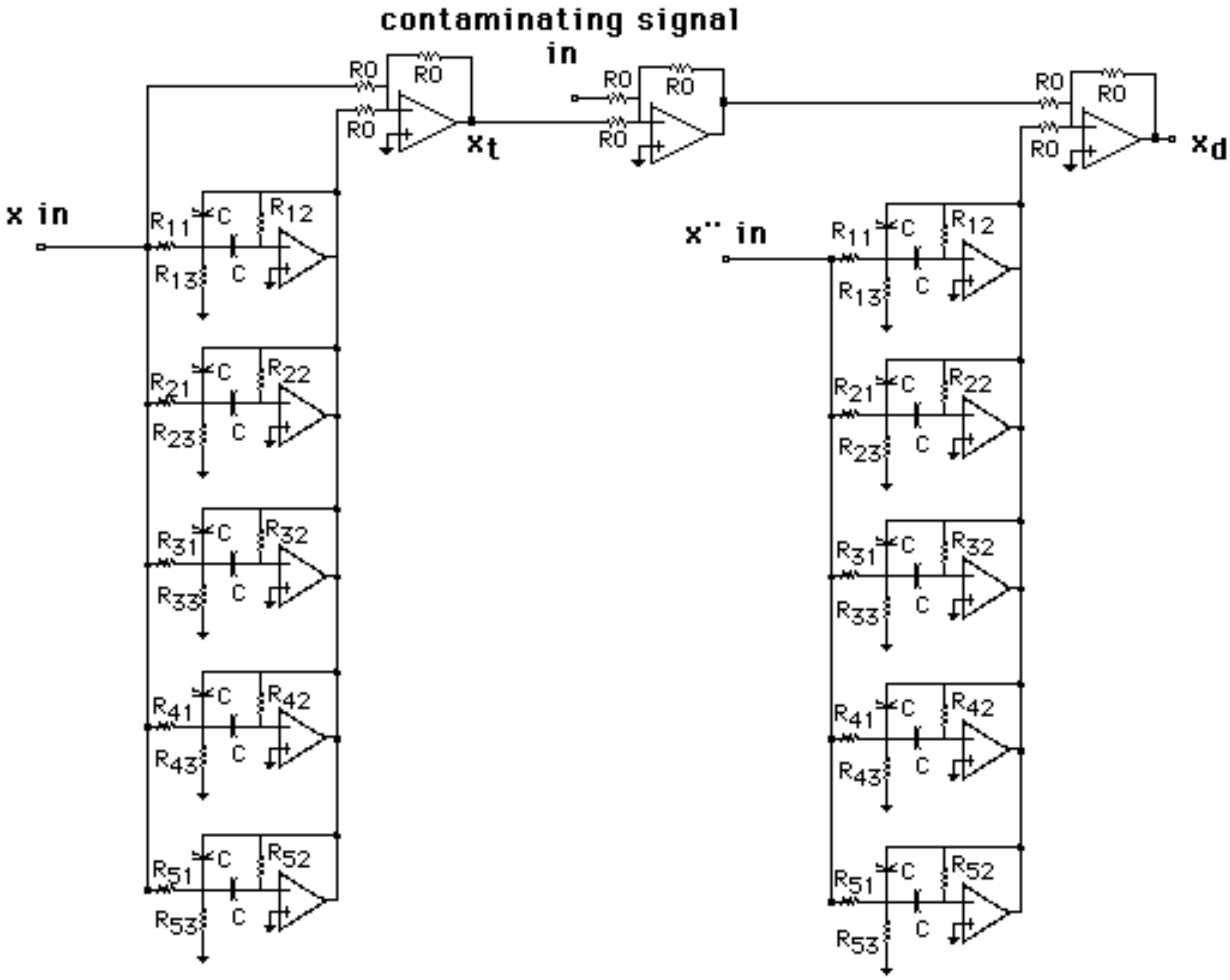
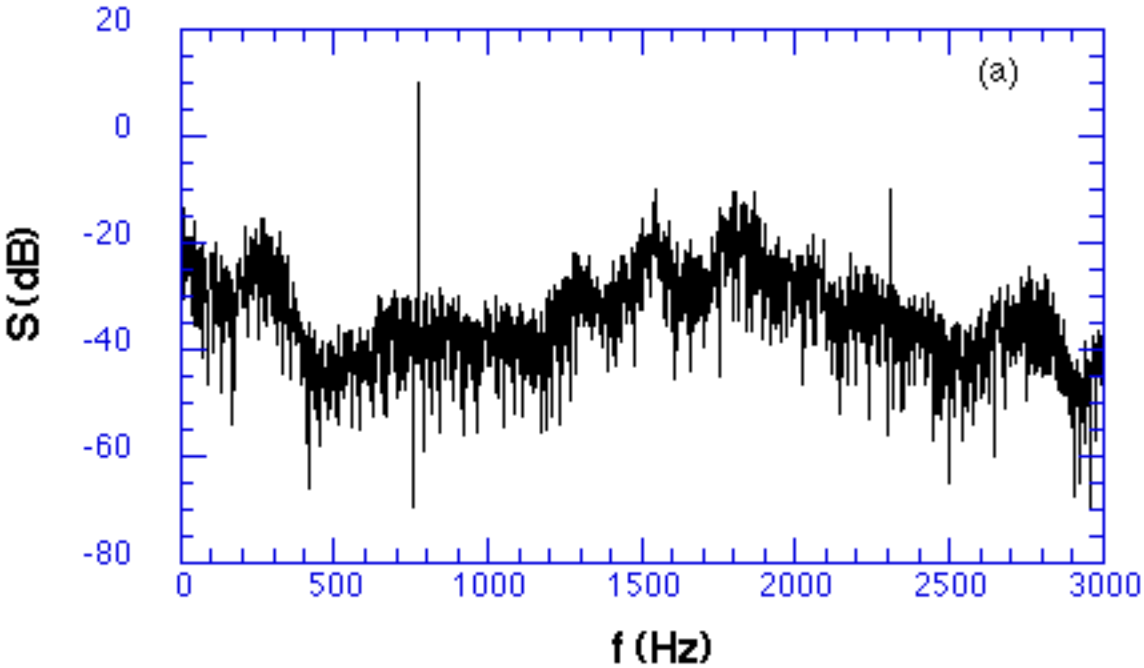


Fig. 2. Schematic of the band stop filters used with the ADF circuit. R0 is 100 kΩ, C = 0.001 μF, and all other resistor values are given in table I.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S(dB)
f (Hz)
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0
-20
-40
-60
-80
0
500
1000
1500
2000
2500
3000

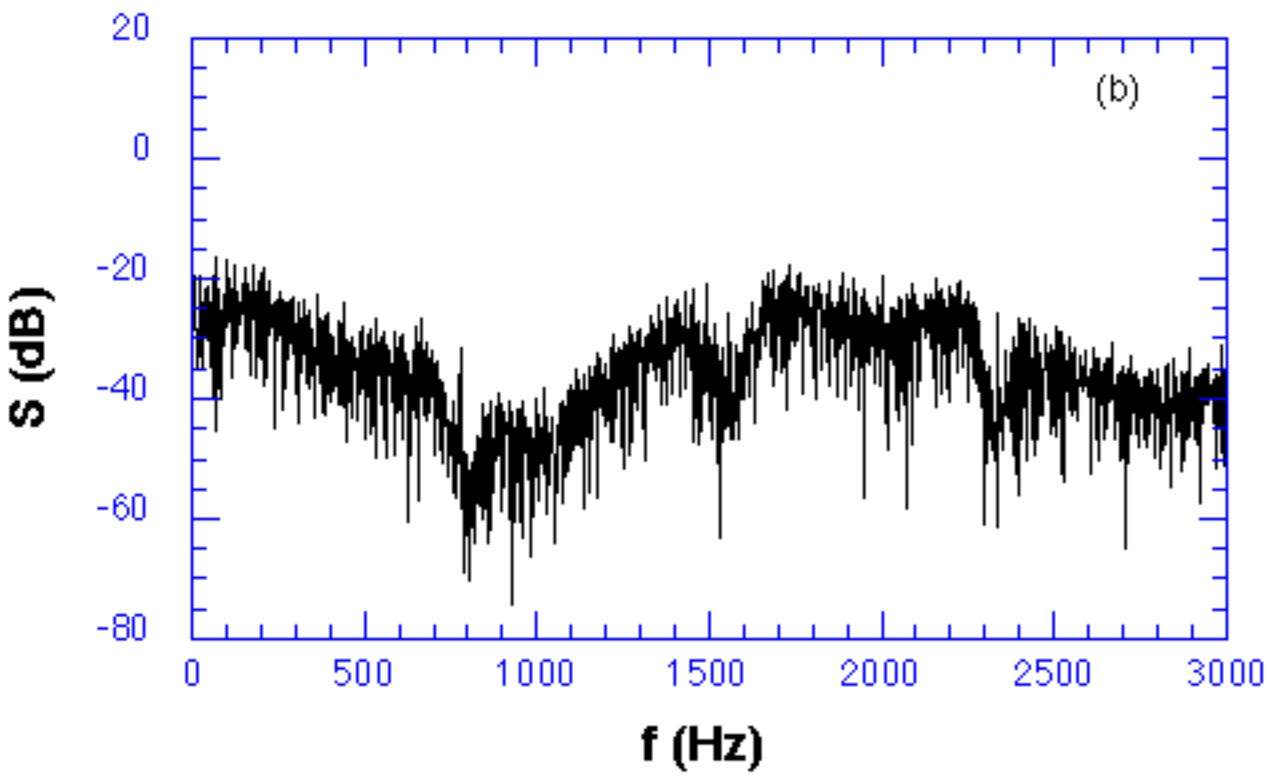

(b)
S (dB)
f (Hz)
20
0
-20
-40
-60
-80
0
500
1000
1500
2000
2500
3000

Fig. 3(a). Power spectrum S vs. frequency f for the x signal from the ADF circuit.

(b) Power spectrum of x_t, the filtered signal that is transmitted between chaotic circuits.

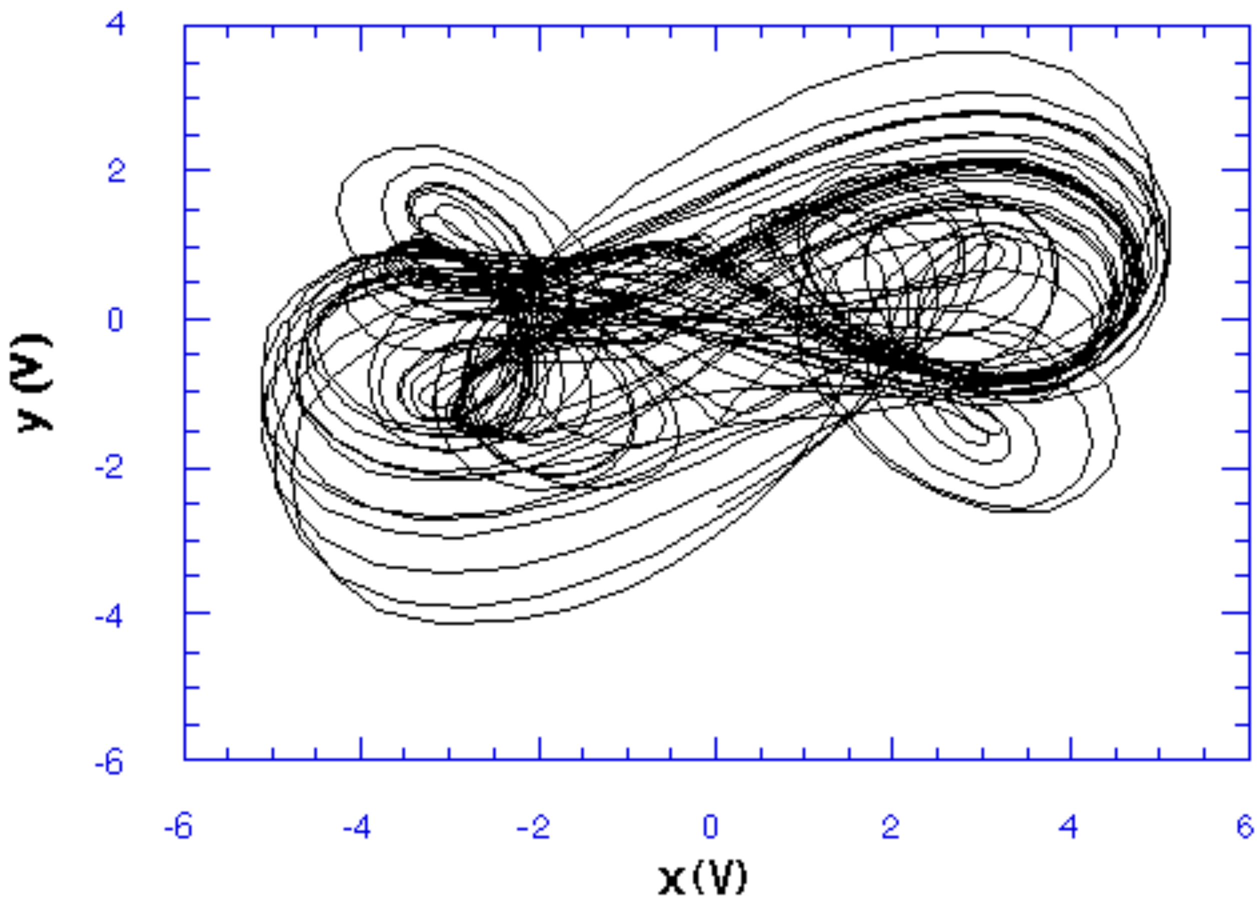


Fig. 4. Attractor for the Augmented Duffing (ADF) circuit of eqs. (5-9).

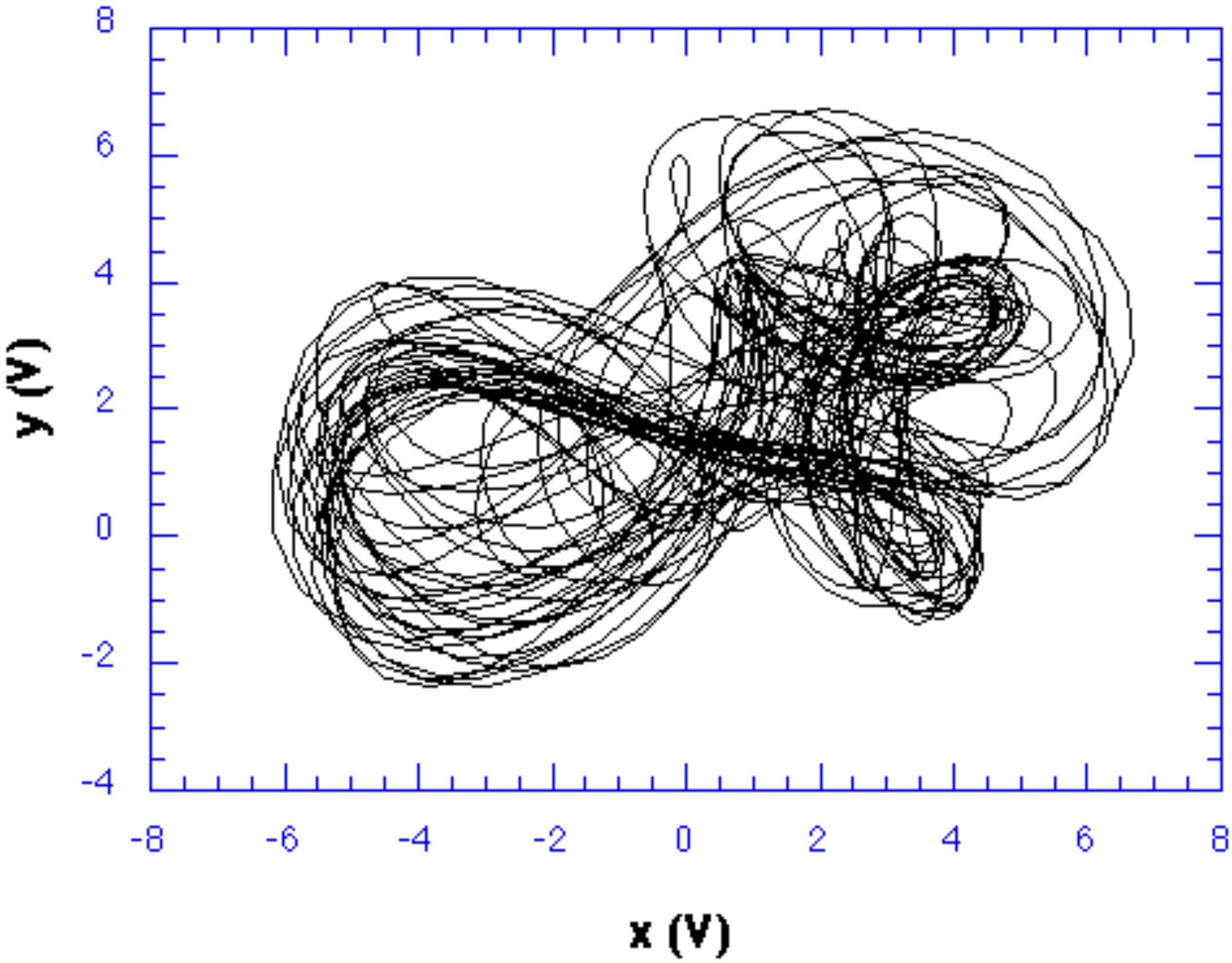
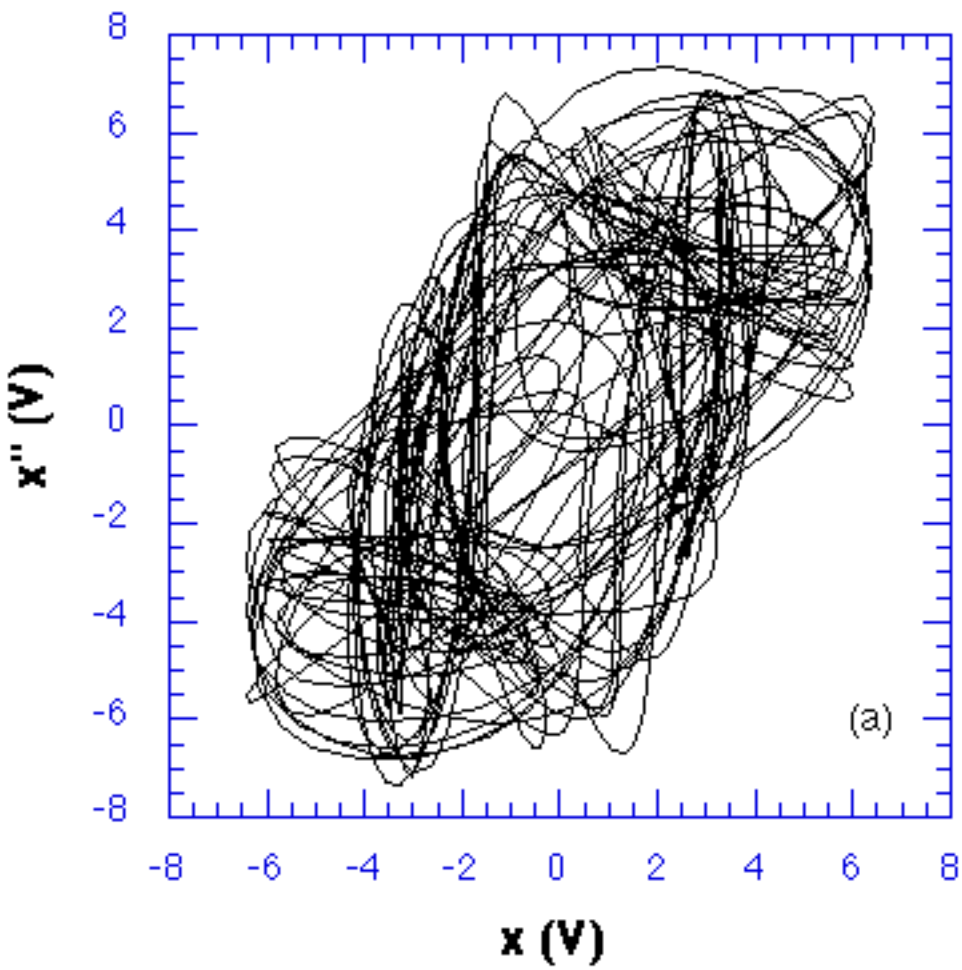


Fig. 5. Attractor for the ADF circuit when the offset A is set to 1.0 V



8
6
4
2
0
-2
-4
-6
-8
x'' (V)
(a)
-8 -6 -4 -2 0 2 4 6 8
x (V)

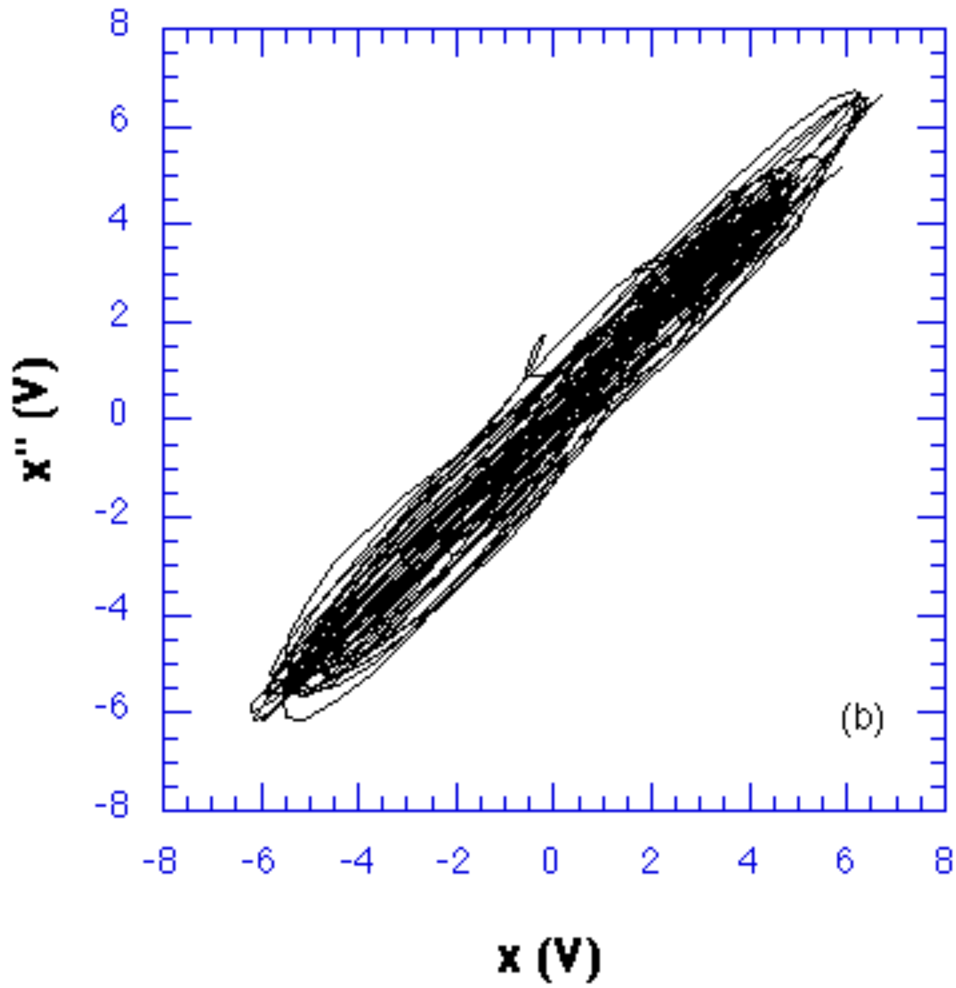


Fig. 6(a). Output signal x" from the response ADF circuit vs. x from the driving circuit when the offset parameter A=0.0 V. (b). Output signal x" from the response ADF circuit vs. x from the driving circuit when the offset parameter A=1.0 V.

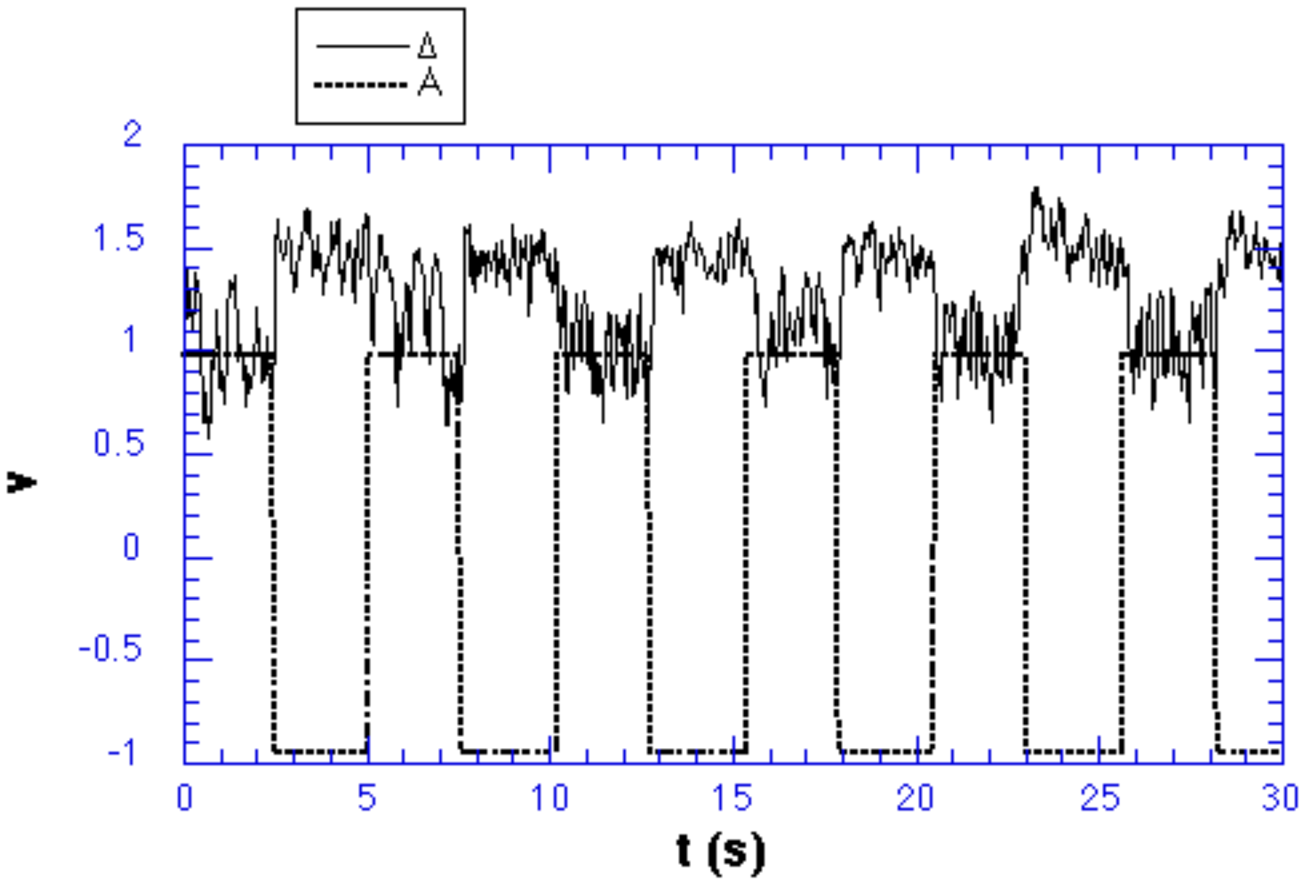


Fig. 7. Offset parameter A in the drive ADF circuit and resulting error signal Δ from the analog controller when A is switched between two values.

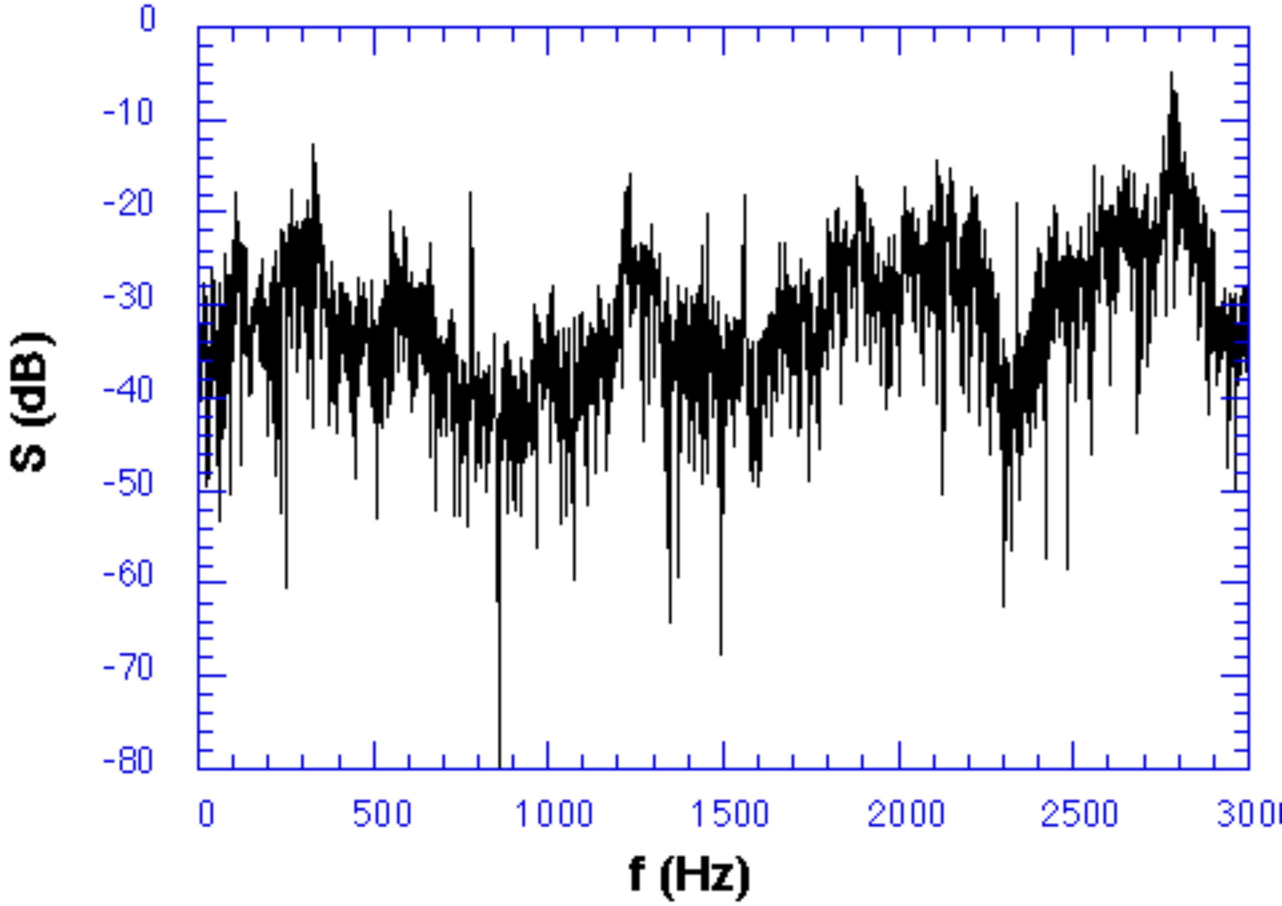


Fig. 8. Power spectrum of ξ_t, the filtered signal from the SWD circuit that is used as a contaminating signal.

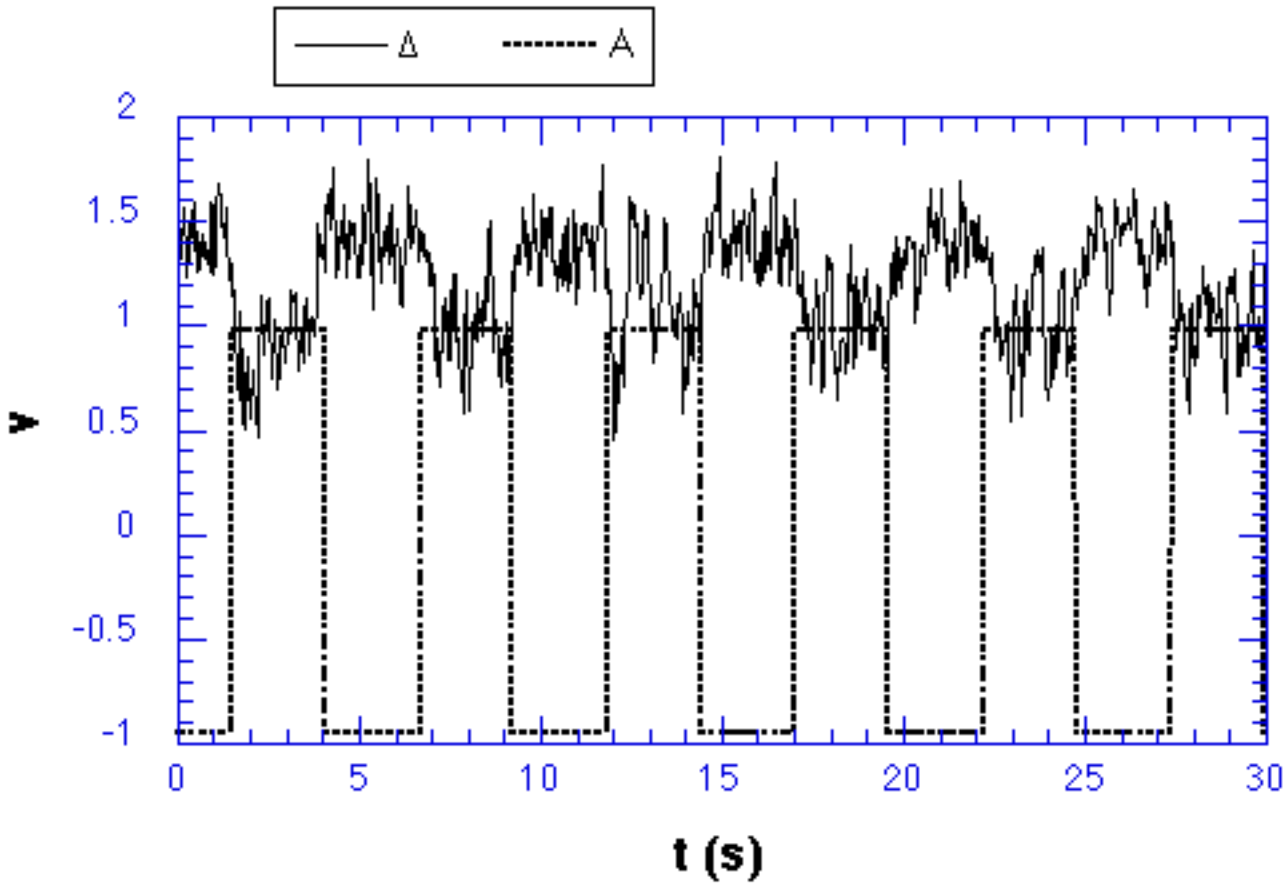


Fig. 9. Offset parameter A in the drive ADF circuit and resulting error signal Δ from the analog controller when A is switched between two values while a contaminating signal (ξ_t) is added to the transmitted x_t signal.

Table I. Resistor values (R_{ij}) for band pass filters.

	j=1	j=2	j=3
i=1	204,000 Ω	408,000 Ω	1026 Ω
i=2	102,000 Ω	204,000 Ω	513 Ω
i=3	68,000 Ω	136,000 Ω	342 Ω
i=4	51,000 Ω	102,000 Ω	256 Ω
i=5	40,800 Ω	82,000 Ω	205 Ω

Table I. Resistor values for bandpass filters.