Spread Spectrum Sequences from Unstable Periodic Orbits

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Abstract -- I use the unstable periodic orbits (UPO's) from a chaotic system to extract a modulated chaotic carrier signal from noise or interference from other chaotic signals. The resulting communication system is similar to CDMA except that the system will still function when interfering chaotic signals are larger than the chaotic carrier signal.

I. Introduction

While much has been written about the use of chaos for communication[1-12], there has been only a little work that actually seeks to use the properties of the chaos itself [5, 7, 9, 11]. The unpredictable nature of chaos has been applied to signal encryption, but as far as actually transmitting and receiving chaotic signals, the broad-band and unpredictable nature of chaos would seem to make it a poor choice for a communications signal. Many versions of self-synchronizing chaotic systems have been proposed, but they are sensitive to noise and use a lot of bandwidth without giving anything in return, such as greater bandwidth efficiency.

I report here some attempts to use properties of chaos that can aid communication. I base this work on unstable periodic orbits. A chaotic attractor is composed of an infinite number of unstable periodic orbits (UPO's), but motion on the attractor may be dominated by a small set of low period orbits [13-20]. If one can recognize which of these UPO's is present at a particular time, one could build a good approximation to the chaotic attractor. In practice, the UPO's are recognizable in a time series taken from a chaotic attractor, so the UPO's form a series of recognizable patterns that can allow one to recognize the attractor. Based on these principles, I construct a UPO receiver.

II General Approach

My approach is this: I first extract a set of low period UPO's from one or more time series generated by a chaotic system. There are several techniques available for this[17, 21], or the orbits may be calculated directly from the equations of motion [22]. I then assemble sequences of these UPO's. I seek to construct all possible sequences of a given length. There may be many possible sequences for a particular length, but most sequences never show up in the actual attractor, so a large number of possible sequences can be discarded. I then compare each of these sequences to a fixed length segment of a time series from the chaotic attractor. Different chaotic systems become uncorrelated with each other, so I compare each unstable periodic orbit sequence with the time series segment by taking the cross-correlation between them. I take the UPO sequence with the highest cross-correlation as the best approximation to the segment from the chaotic time series. The fit is never exact because noise (including roundoff error) causes the chaotic attractor never to be exactly on a UPO, and the chaotic attractor may move to a different UPO at some unpredictable time.

In a previous work [23] I used this method to do some simple fits to time series from maps and flows. In this work, I refine the techniques for construction of the UPO sequences, and I characterize the UPO fits in ways appropriate for communication applications.

The work presented here is similar to direct sequence spread spectrum [24, 25], and there has been some other work in that area. Mazzini et al. [7, 9] studied systems based on the chaotic shift map, and found that a chaotic spread spectrum system could achieve performance as good as that as a system using Gold codes. Yang and Chua [11] studied a similar system. In both cases, the sequences from chaotic maps were quantized and truncated, so that the sequences were actually periodic.

III Constructing UPO sequences

The process of constructing UPO sequences begins with isolating the UPO's. Typically, I isolate UPO's up to period 16, although isolating more or fewer may not greatly affect the approximation. Reference [23] shows some typical UPO's for the Lorenz equations. All possible sequences for this set of UPO's are then constructed up to some maximum sequence length. It is necessary in this construction to decide which UPO's can follow which other UPO's. For the work in this paper, the distance from the end of 1 UPO to the beginning of the next determined which orbits could go in series. The maximum distance allowed between each component of the end of 1 orbit and the beginning of the next was orbits was 2% of the RMS amplitude of that component of the chaotic attractor. This distance could also be adjusted to change the approximation. All possible phases of each UPO were used.

For a first pass, I created sequences about 2 cycles long (128 points at the sampling rate I used). These 2 cycle UPO sequences were the basic building blocks of the approximation. To create longer UPO sequences, rather than go through the prediction step again, these 2 cycle (128 point) sequences were simply combined sequentially to yield 4 cycle (256 point) sequences. There were a large number of 4 cycle sequences, and many of these sequences contained large discontinuities or did not correspond to physically realizable sequences. To reduce the number of sequences, I then generated a long time series from the original chaotic attractor, and measured the cross correlation between each 4 cycle UPO sequence and a 4 cycle segment from the chaotic time series. The 4 cycle UPO sequence with the largest cross correlation was chosen as the best fit to the time series segment. This process was repeated for a large number of different time series segments, on the order of 10,000 to 100,000 total segments.

From the data on which 4 cycle UPO sequence was the best fit to each time series segment, a histogram was made. Some UPO sequences showed up more often than others, and some never showed up. All UPO sequences that occurred fewer than some threshold number of times were discarded, so that the total number of UPO sequences was reduced by a factor of 4 to 10. Excluding too many UPO sequences degraded the cross-correlations noticably, so the threshold was set so that the cross-correlations were not degraded by too much.

This truncation of the set of UPO sequences worked because many of the UPO sequences never actually occurred in the chaotic attractor (there has been some work on creating "grammars" [26, 27] which describe what sequences of UPO's can occur in a chaotic attractor). In addition, some sequences were very similar to each other, so substituting one sequence for another only slightly reduced the cross correlation.

These same procedures could be repeated to create UPO sequences of greater lengths.

IV Application to a chaotic flow

One of the chaotic systems defined by Sprott is used here. Sprott compiled a list of 20 different chaotic flows [28]. The system used here is a modified version of Sprott's system B, and is defined by:

$$\begin{aligned}\frac{dx}{dt} &= 0.4yz \\ \frac{dy}{dt} &= x - 1.2y \\ \frac{dz}{dt} &= 1 - xy\end{aligned} \tag{1}$$

Figure 1 shows the attractor for this system.

The z signal is used as a carrier signal. The UPO's up to period 16 were extracted from a 50,000 point time series generated by eq. (1) with a time step of 0.2. All 3 components were used to find the UPO's. All possible pairs of UPO's were combined using the distance criteria defined above and the resulting sequences were truncated at 128 points (about 2 cycles), resulting in over 29,000 possible sequences.

The z component of these UPO sequences was compared to a long time series from eq. (1) and a histogram was created based on which UPO sequences had the highest cross correlation with the segments of length 128 points from the time series. Based on the histogram, a set of 108 UPO sequences that occurred most often was chosen.

These 108 UPO sequences were then combined to create 108×108 = 11,664 UPO sequences of 256 points. The process of comparing to a time series signal and creating a histogram was again followed to select a set of 1162 UPO sequences of 256 points (about 4 cycles). By the same process, a set of 8616 UPO sequences of length 512 (about 8 cycles) was also created. The memory required to store these sequences did not increase greatly with sequence length because the same 128 point building blocks were used.

To encode information, the z signal was multiplied by $s = \pm 1$. Other modulation techniques could also have been used, but this was an easy technique to implement and test. The modulated z signal was then cross correlated with a set of 128, 256 or 512 point UPO sequences to extract the value of s . The value of s varied every 128 points if 128 point UPO sequences were used for comparison, 256 points of 256 point UPO sequences were used, etc.

I first characterized the performance of the UPO receiver when Gaussian noise was added. I numerically calculated the probability of bit error (P_b) as a function of energy per bit (E_b) divided by noise power spectral density (N_0). The result of this calculation is shown in Fig. 2 (the UPO sequence length was 256 points). Achieving a particular probability of bit error required 3-5 dB more energy per bit than some common digital modulation techniques listed in Sklar [25], but no synchronization is required, so the UPO receiver may be less sensitive to changing channel conditions.

V Interference from Other Chaotic Signals

A greater problem in a multiuser communication system is separating the desired carrier signal from carrier signals generated by other users. A chaotic signal from another one of Sprott's systems, a modified version of system C [28], was used to test the effect on the UPO receiver of interference from another user. Sprott's system C was very similar to Sprott's system B (eq. (1)):

$$\begin{aligned}\frac{dx}{dt} &= \alpha(0.4yz)\\ \frac{dy}{dt} &= \alpha(x - 1.2y)\\ \frac{dz}{dt} &= \alpha\left(1 - x^2\right)\end{aligned} \tag{2}$$

The time constant α was included so that the peak frequencies in the power spectra from the z signals from eqs. (1) and (2) would match. The value of α was 0.75. Figure 3(a) shows the power spectrum for the z signal from eq. (1), while Fig. 3(b) shows the power spectrum from the z signal of eq. (2).

Figure 4 shows the probability of bit error (P_b) for the UPO receiver as a function of the ratio of the RMS amplitude of the signal from Sprott's system C (the interfering signal) to the RMS amplitude of the signal from Sprott's system B (the carrier signal) (R_A). When 128 point UPO sequences are used and the RMS amplitudes are equal, the probability of a bit error is about 25%. When 256 point UPO sequences are used, the probability of bit error is about 8%, dropping to 3.5% when 512 point sequences were used. This decrease in bit error rate does demonstrate some bandwidth efficiency- the greater the bandwidth (longer the UPO sequence), the lower the interference from other users.

Figure 4 also shows another important feature of the UPO receiver. A conventional code division multiple access (CDMA) receiver will not work if the signal from an interfering user is larger than the signal of interest [24]. Complicated power management techniques are necessary to correct for this problem. The UPO receiver detects the message from the proper user, even when the signal from another user is larger. The conventional CDMA receiver depends on synchronizing to the proper signal, after which other signals are rejected because they are not synchronized; the UPO receiver in this paper separates signals based on lack of correlation and a difference in structure between signals from different transmitters.

Mazzini et al,[7, 9] and Yang and Chua [11] are able to calculate the number of possible users for a multiuser communications system whose spreading signal consists of the output from a 1-d chaotic map. The map output is quantized and truncated to a finite length, so the output sequence is periodic. The calculations in those papers depended on the fact that the probability distributions of both the interfering and spreading signals were Gaussian. In this work, neither signal is Gaussian, so such a calculation is not possible. At best, a simple estimate is possible. Assuming the different chaotic carrier signals add in a random fashion (signals from uncoupled chaotic systems will never be in phase), then the probability of bit error should increase approximately as the square root of the number of interferers. From this point, it is necessary to estimate how similar we would like the chaotic systems to be to each other, and how many such chaotic systems may be built. Different modulation techniques might also increase bandwidth efficiency.

It is possible to test the UPO receiver with chaotic interference that is not much like the chaotic carrier. The two Sprott systems in eqs. (1-2) are very similar to each other, so they should cause a large amount of interference with each other. To examine the effect of interference from a chaotic system that is not much like Sprott's system B of eq. (1), a Lorenz [29] system was used. The Lorenz system was described by:

$$\begin{aligned}\frac{dx}{dt} &= \alpha\left[16(y-x)\right] \\ \frac{dy}{dt} &= \alpha\left[-xz+45.92x-y\right] \\ \frac{dz}{dt} &= \alpha\left[xy-4z\right]\end{aligned} \tag{3}$$

The time constant α was used to match the time scale of eq. (3) to eq. (1). The value of α was 0.05. Eq. (3) was integrated with a time step of 0.2.

Figure 5 shows what happened when the z signal from eq. (3) (with DC component removed) was added as an interfering signal to the modulated z signal from eq. (1). Figure 5, shows the probability of bit error (for a 256 point UPO sequence) as a function of the ratio of the RMS amplitude of the interfering signal to the RMS amplitude of the chaotic carrier signal (R_A), when the interfering signal is from either the Lorenz system (eq. (3)) or Sprott's system C (eq. (2)) . For $R_A < 1$, the probability of bit error is lower when the interfering signal comes from the Lorenz system, but for $R_A > 1$, the probability of bit error is about the same for either interfering signal. Figure 5 does show that there is some advantage to using chaotic systems that are not very similar to each other.

VI Possible improvements

One major disadvantage of the UPO receiver described in this paper is that it is necessary to search through a large number of UPO sequences to find the best fit. Some improvements are possible in this searching time. In order to find the probability of bit error for 512 point sequences in Fig. 4, the UPO sequences were arranged in order of their probability, so that the most probable UPO sequences were searched first. Before searching, the maximum possible correlation was first estimated for each noise level. The search for the UPO sequence with the largest possible cross-correlation proceeded only until a sequence was found whose cross-correlation with the incoming time series was greater than 95% of the maximum estimated cross correlation. This sequence was taken as the best fit. Proceeding in this manner, on the average less than 10% of the sequences were searched.

For the numerical examples in this work, it was possible to estimate the largest possible cross-correlation by comparing a noise-contaminated UPO sequence with a noise free copy of the same sequence. In a real receiver, the sequence being transmitted is not known ahead of time, unless some arrangement is made to transmit a known UPO sequence at a known time for noise estimation purposes. Otherwise, some other method must be found to estimate the noise.

VII Conclusions

It is possible to use unstable periodic orbits of a chaotic system to extract a signal from that system from large amounts of noise. In this work, I have done the signal separation using flows; in other work, I have also applied these methods to maps [23]. While these methods overcome some of the disadvantages of conventional CDMA (power management is not necessary), there are still obstacles to be overcome before a UPO receiver could be practical.

The primary disadvantage with a UPO receiver is the large number of UPO sequences that must be searched. I have shown above one technique that can reduce the number of sequences searched. It might also be possible to combine the UPO receiver with a transmitter based on the work of Hayes [5], in which only specific UPO sequences are sent.

Estimating bandwidth efficiency is also difficult for the UPO receiver because it is difficult to estimate the number of different chaotic systems that can be designed. Sprott [28] lists 20 different chaotic flows of three dimensions, so it should be possible to design many chaotic systems. Different modulation techniques might also increase bandwidth efficiency at the expense of making the transmitter more complex.

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Figure Captions

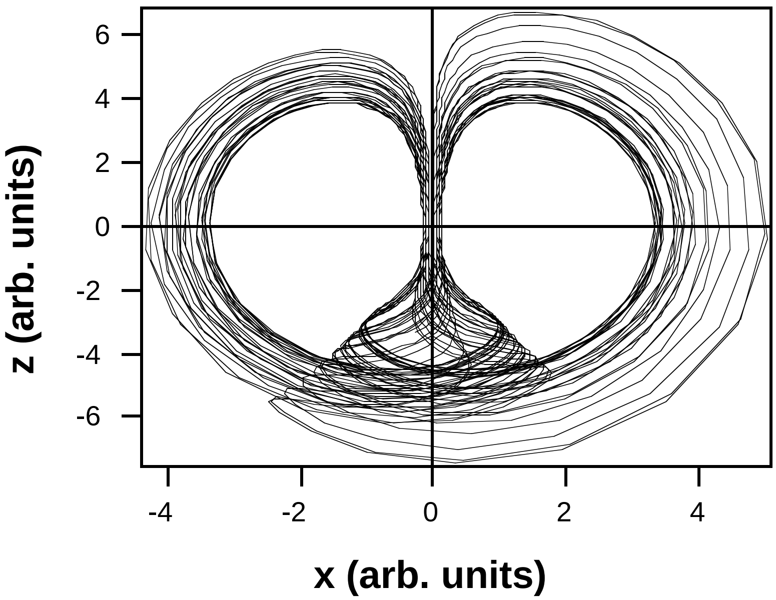

Figure 1. Attractor for Sprott's system B of eq. (1).

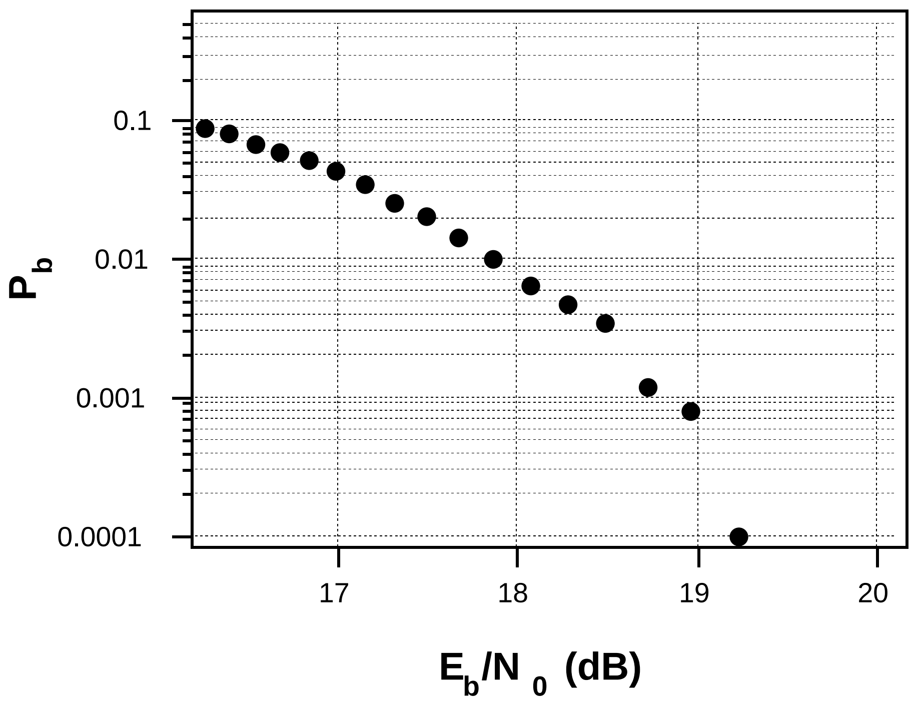

Figure 2. Probability of bit error (P_b) as a function of energy per bit (E_b) divided by noise power spectral density (N_0) when Gaussian noise is added to a binary modulated signal from Sprott's system B (eq. (1). The length of the UPO sequences was 256 points (4 cycles).

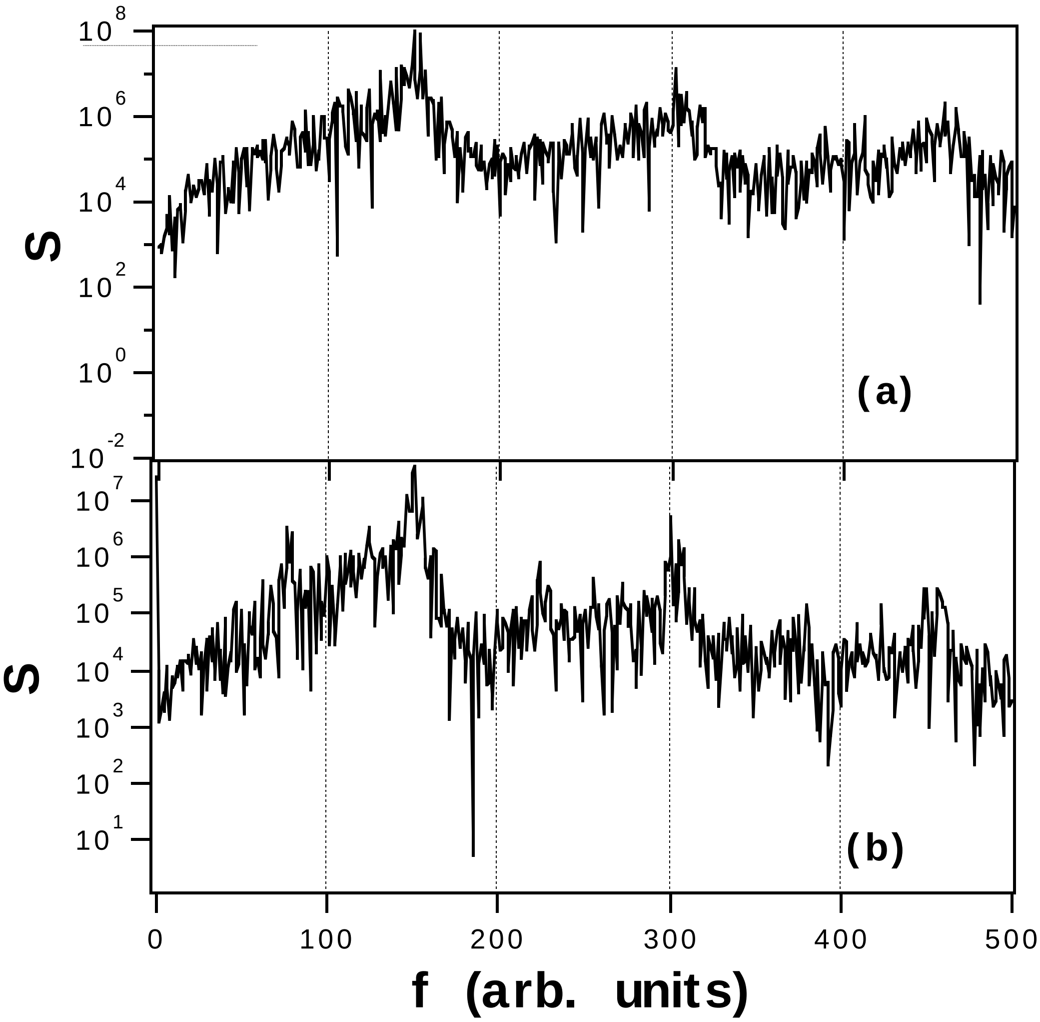

Figure 3. (a) Power spectrum of carrier signal from Sprott's system B of eq. (1). (b) Power spectrum of interfering signal from Sprott's system C of eq. (2).

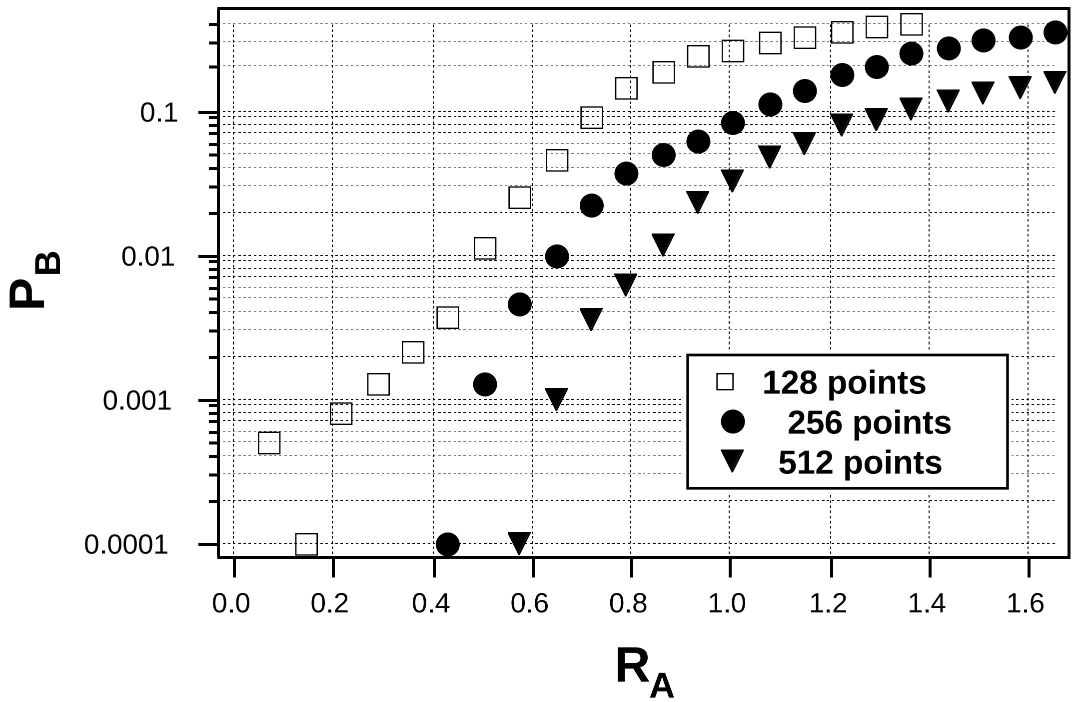

Figure 4. Probability of bit error (P_b) as a function of R_A, the ratio of the RMS amplitudes of the interfering chaotic signal to the carrier chaotic signal. The open squares are for when the UPO sequence length was 128 points, the closed circles for a UPO sequence length of 256 points, and the upside-down triangles for a UPO sequence length of 512 points.

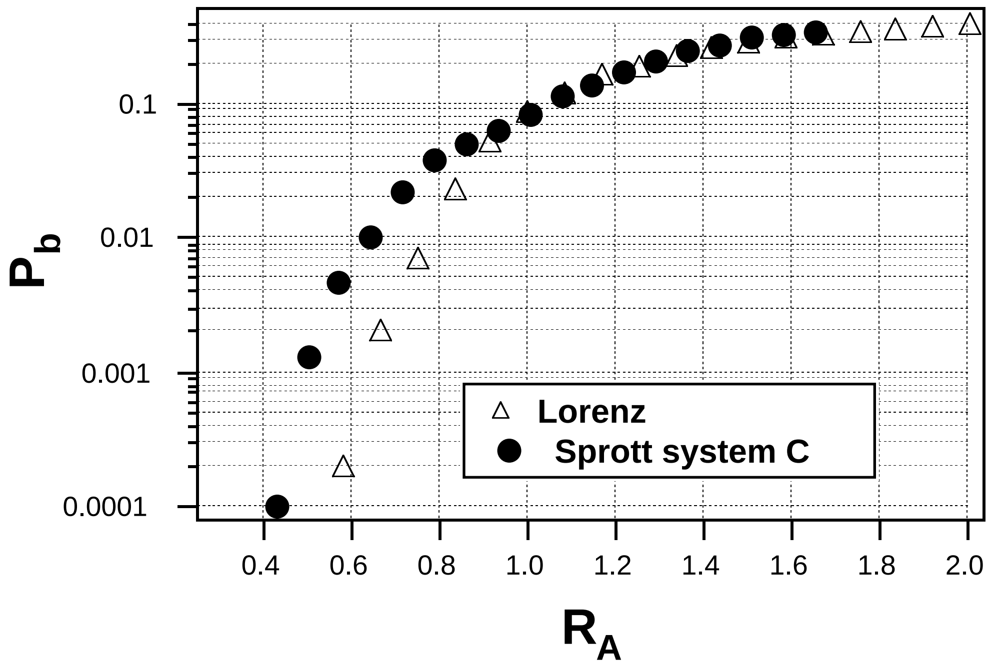

Figure 5. Probability of bit error (P_b) as a function of R_A, the ratio of the RMS amplitudes of the interfering chaotic signal to the carrier chaotic signal. The closed circles are for when the interfering signal came from Sprott's system C (as in Fig. 4), while the open triangles are for when the interfering signal came from the Lorenz system of eq. (3).