Synchronizing Nonautonomous Chaotic Circuits

Thomas L. Carroll and Louis M. Pecora
Code 6341, Naval Research Lab, Washington, DC 20375

ABSTRACT

In this paper we extend the techniques of chaotic synchronization to periodically forced systems. The periodic forcing contributes a zero Lyapunov exponent, which must be accounted for. An advantage of nonautonomous synchronization is that the synchronization is very noise resistant.

1. Introduction

In the form of chaotic synchronization demonstrated by us[1-4] and others[5-10], one reproduces a part of an autonomous chaotic system. This subsystem is driven by a signal from the full chaotic subsystem. When all the Lyapunov exponents of the driven subsystem (which we call the response system) are less than 0, the response system will synchronize with the drive system. Signals in the response system will be the same as the corresponding signals in the drive system, assuming both systems start in the same basin of attraction. By cascading subsystems[4], it is possible for the response system to reproduce the drive signal, allowing for different kinds of signal modulation.

In this paper, we extend these techniques to chaotic systems that are are nonautonomous (they have some explicit time dependence) This work has been previously published[11]. We use a chaotic system that contains a sinusoidal forcing term[12]. If one cascades subsystems of this chaotic system so that the chaotic driving signal is reproduced, then one of the cascaded subsystems must include the sinusoidal forcing. This forcing term gives the response system a zero Lyapunov exponent because the phase of the response will maintain a constant relation to the phase of the drive. If the two phases are not equal, synchronization will not occur. In this paper, we show how to correct the phase of the forcing term in the response so that it matches the phase of the forcing term in the drive, allowing the response system to synchronize with the drive system. The use of periodically forced chaotic systems in synchronization has the advantages of being far less sensitive to noise than autonomous systems and more like the types of synchronizing systems that are already used in communications and control.

2. Synchronization

The theory of the synchronization of chaotic systems has been reported in detail elsewhere[1], so we present only a brief description here. We begin with a dynamical system that may be described by the ordinary differential equation

$$\dot{u}(t) = f(u) \tag{1}$$

The system is then divided into two subsystems, u= (v,w);

$$\begin{aligned}\dot{v} &= g(v,w)\\ \dot{w} &= h(v,w)\end{aligned} \tag{2}$$

where $v=(u_1,...,u_m)$, $g=(f_1(u),...,f_m(u))$, $w=(u_{m+1},...,u_n)$, and $h=(f_{m+1}(u),...,f_n(u))$. The division is truly arbitrary since the reording of the u_i variables before assigning them to v, w, g, and h is allowed.

Now create a first response system by duplicating a new sub-system w’ identical to the w system, substitute the set of variables v for the corresponding v’ in the function h, and augment Eqs. (2) with this new system, giving,

$$\dot{v} = g(v,w)$$
$$\dot{w} = h(v,w)$$
$$\dot{w}' = h(v,w'). \tag{3}$$

If all the Lyapunov exponents of the w' system (as it is driven) are less than zero, then $w' - w \rightarrow 0$ as $t \rightarrow \infty$.

It is possible to take this system further. One may also reproduce the v subsystem and drive it with the w' variable[3, 4]. If all the Lyapunov exponents of this v'' subsystem are less than 0, then $v'' \rightarrow v$ as $t \rightarrow \infty$.

3. Nonautonomous Systems

Periodic forcing contributes a zero exponent to one of the subsystems. We show here how to compensate for this zero exponent to synchronize nonautonomous chaotic systems.

We use for an example a previously described circuit[11]. This circuit may be described by the equations:

$$\frac{dx}{dt} = \beta(y - z) \tag{4}$$

$$\frac{dy}{dt} = \beta\left(-\Gamma_y y - g(x) + \alpha\cos(\omega t)\right) \tag{5}$$

$$\frac{dz}{dt} = \beta\left(f(x) - \Gamma_z z\right) \tag{6}$$

$$g(x) = -3.8 + \left(\frac{1}{2}\right)\left(|x + 2.6| + |x - 2.6| + |x + 1.2| + |x - 1.2|\right) \tag{7}$$

$$f(x) = \frac{1}{2}x + |x - 1| - |x + 1| \tag{8}$$

where g(x) and f(x) are piecewise linear functions. The constants were: $\alpha = 2.0$, $\Gamma_y = 0.2$, $\Gamma_z = 0.1$, the time factor β is $10^4\ s^{-1}$ and the frequency ω is $2\pi f_d$, where the forcing frequency f_d is 769 Hz. The circuit for g(x) was shown in reference 13, while the circuit for f(x) is quite similar. Both g(x) and f(x) are based on diode function generators[14, 15].

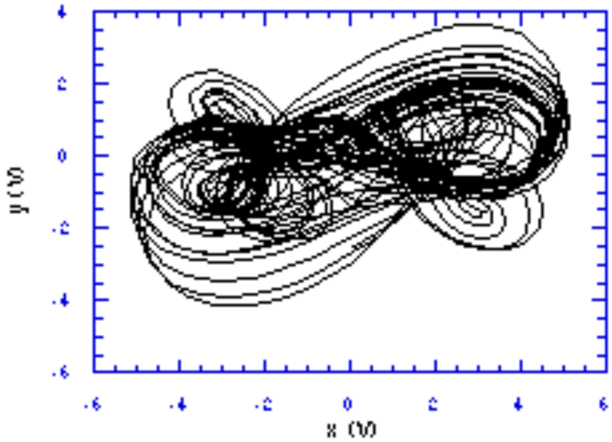

Figure 1. Plot of y voltage vs. x voltage from the chaotic circuit.

Equations (4) and (5) by themselves are a 1-well Dufing oscillator[12]. For these parameter settings, the Duffing oscillator would be in a period-one attractor. Equation (6) is added to make the system chaotic. If the feedback loop between eqs. (6) and (4) is cut, the system will have a largest exponent of 0, corresponding to the sinusoidal forcing term. This makes as an obvious choice for a response system the configuration described by these equations:

$$\frac{dz'}{dt} = \beta\left(f(x) - \Gamma_z z'\right) \tag{9}$$

$$\frac{dx''}{dt} = \beta\left(y'' - z' \right) \tag{10}$$

$$\frac{dy''}{dt} = \beta\left(-\Gamma_y y'' - g(x'') + \alpha\cos\left(\omega_r t + \phi_r\right)\right) \tag{11}$$

with the x signal used as a drive The parameter ϕ_r is the difference in phase between the sinusoidal forcing in the response and the drive.

Figure 1 is a plot of y vs x from the driving circuit. The Lyapunov exponents for the drive system, calculated from eqs. (4-8) (with β = 1.0 and ω_d = 0.42223) by the method of Eckmann and Ruelle[16], were 0.06, -0.14 and -0.22. The sinusoidal forcing term is treated as a parameter in this calculation, so its 0 exponent does not show up here. The Lyapunov exponents for the response system (calculated from eqs. (9-11)) were -0.07, -0.10 and -0.19. Once again, the sinusoidal forcing was treated as a parameter, so the zero exponent does not show up here. The negative exponents show that if the parameter ϕ_r is zero, the response circuit described by eqs. (9-11) will synchronize with the drive circuit described by eqs. (4-8).

The chaotic x driving signal does carry enough information to correct the value of ϕ_r . To extract this information, we used a strobe signal consisting of either the output signal from the response circuit x'' or the sinusoidal forcing term for the response circuit.

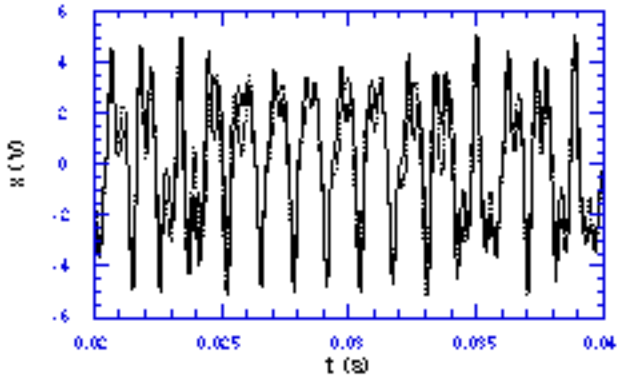

Figure 2. Drive voltage x (solid line) and synchronized response voltage x" (dotted line) from the synchronized nonautonomous circuit.

The value of x was held by a sample and hold circuit when the strobe signal crossed zero in the positive direction. The output of the sample and hold circuit was integrated by an analog integrator with a time constant of approximately 10 s to generate a phase error signal. This same type of error detector was used in a previous chaotic circuit[3].

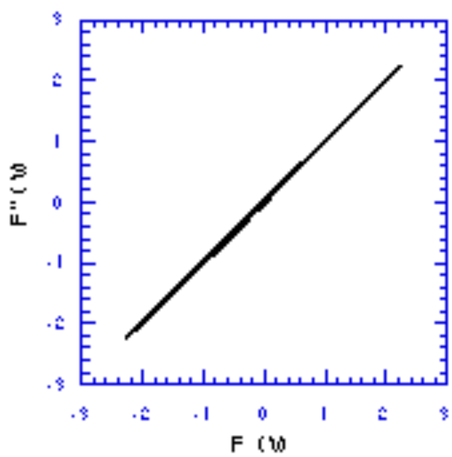

Figure 3. Periodic forcing F" for the synchronized response circuit vs periodic forcing F for the drive circuit.

To close the feedback loop, the phase error signal was multiplied by 0.01 and used to modulate the frequency of the function generator providing the sinusoidal forcing for the response circuit. When the frequency of the forcing for the response circuit was within about 2 Hz of the frequency of the forcing for the drive circuit, the feedback loop forced the phase difference ϕ_r to zero, and the two circuits were synchronized. Without the feedback loop, the response phase drifted relative to the drive phase. Figure 2 shows digitized time series of x and x" from the circuit when synchronization was achieved. Figure 3 shows a plot of the output of the forcing for the response circuit vs. the forcing for the drive circuit.

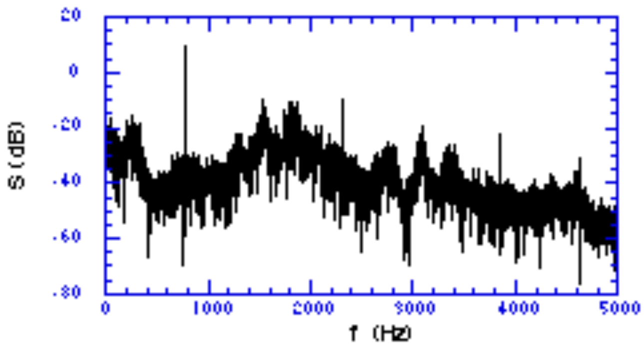

Figure 4. Power spectrum of chaotic drive signal.

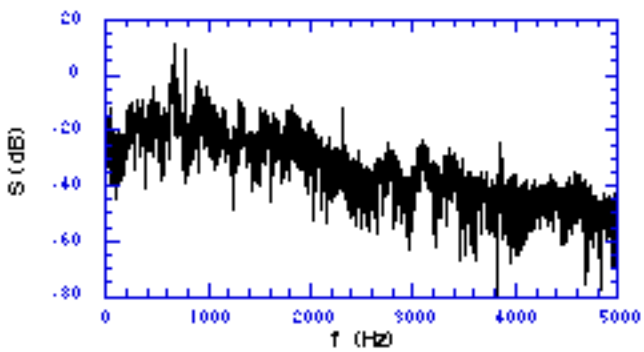

Figure 5. Power spectrum of the chaotic drive signal when another chaotic signal has been added.

When x" was used as a strobe signal, the synchronized chaotic circuits maintained synchronization even when large amounts of noise were added to the driving signal x. We found this to be true both for white noise and for chaos. Figure 4 is a power spectrum of the x driving signal. Figure 5 is the power spectrum of the driving signal when large amplitude chaos from a chaotic circuit[3] has been added. The added chaotic signal was amplified so that its amplitude was about twice the amplitude of the drive signal. The main frequencies in the added chaotic signal were close to the main frequencies in the driving signal. Figure 6 is an plot of the sinusoidal forcing for the response circuit vs. the forcing for the drive circuit when this chaos was added to the drive signal. There is an offset between the phases, but there is no drift. If the x driving signal is removed and only the added chaotic signal is present, the phase of the response forcing will not remain constant relative to the phase of the drive forcing.

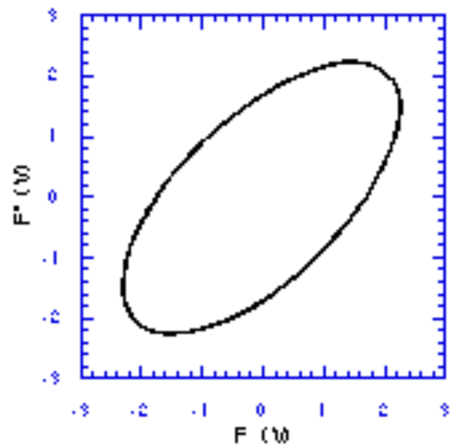

Figure 6. Periodic forcing F" for the synchronized response circuit when chaos is added to the drive signal vs. forcing F for the drive circuit.

4. Conclusions

It is possible to synchronize periodically forced chaotic circuits by using a feedback device to correct the phase of the periodic forcing in the response system. This is equivalent to making the zero Lyapunov exponent in the response system negative.

This procedure creates new possibilities for the use of synchronized chaotic circuits. It was shown above that these synchronized chaotic circuits may be used to detect the presence of a chaotic signal in real time, even when it is added to another chaotic signal that overlaps the drive signal in frequency. We plan to find out if this will work when the added signal is from another nonautonomous circuit driven at the same frequency. This would allow multiple communications signals to be sent in the same frequency band.

We have also not discussed other ways of extracting phase information from the x driving signal, such as taking a long time average of the product of the x drive signal and the periodic forcing term for the response circuit. This average product will be maximized (in this circuit) when the drive and response forcing terms are in phase. The average product will also have some specific amplitude when the two circuits are in phase. Knowledge of this amplitude will allow the drive signal to be set to the proper amplitude if its amplitude has been altered. This could be a concern in noisy environments. In general, periodically forced synchronized chaotic circuits are similar to common systems in communications and control, and so may be easier to apply to real problems.

5. References

1. L. M. Pecora and T. L. Carroll, *Phys. Rev. A* **44** (1991) 2374.
2. T. L. Carroll and L. M. Pecora, *IEEE Trans. CAS* **38** (1991) 453.
3. T. L. Carroll and L. M. Pecora, *Int. J. Bifurc. and Chaos* **2** (1992) 659.
4. T. L. Carroll and L. M. Pecora, *Physica D* **67** (1993) 126.
5. V. S. Anishchenko, T. E. Vadivosa, D. E. Postnov and M. A. Safonova, *Int. J. Bifurc. and Chaos* **2** (1992) 633.
6. K. M. Cuomo and A. V. Oppenheim, *Phys. Rev. Lett.* **71** (1993) 65.
7. M. de Sousa Viera, A. J. Lichtenberg and M. A. Lieberman, *Phys. Rev. A* **46** (1992) R7359.
8. L. O. Chua, L. Kocarev, K. Eckart and M. Itoh, *Int. J. Bifurc. and Chaos* **2** (1992) 705.

9. L. Kocarev, K. S. Halle, K. Eckert, L. O. Chua and U. Parlitz, *Int. J. Bifurc. and Chaos* **2** (1992) 709.
10. U. Parlitz, L. O. Chua, L. Kocarev, K. S. Halle and A. Shang, *Int. J. Bifurc. and Chaos* **2** (1992) 973.
11. T. L. Carroll and L. M. Pecora, *IEEE Trans. Circuits Syst.* **in press** (1993)
12. C. Hayashi, *Nonlinear Oscillations in Physical Systems* , (Princeton University Press, Princeton, NJ, 1964)
13. T. L. Carroll and L. M. Pecora, *Physical Review E in press* (1993)
14. D. H. Sheingold, *Nonlinear Circuits Handbook* , (Analog Devices, Norwood, MA, 1976)
15. C. L. Johnson, *Analog Computer Techniques* , (McGraw-Hill, New York, 1963)
16. J. Eckmann and D. Ruelle, *Reviews of Modern Physics* **57** (1985) 617.