Hard to Detect Chaotic Communications

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Abstract

Work on self-synchronizing systems for communications has had limited practicality because the chaotic signals were not as easy to detect in the presence of noise as conventional spread-spectrum signals. This difficulty may actually be an advantage in some cases, where one wants to conceal the existence of the communications signal. Conventional communications signals are cyclostationary; while they may look random, they have statistical properties that vary periodically. One may design chaotic communication signals that lack this cyclostationary property, and therefore are harder to detect.

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I. INTRODUCTION

While there has been much study of chaos applied to communications [1–13], in many cases the chaos is more of a burden then an asset. Using chaotic signals instead of periodic signals, or using self-synchronizing receivers instead of stored reference receivers, usually results in signals that are harder to detect than conventional periodic or spread spectrum signals.

There are some situations where one wants a signal that is hard for an eavesdropper to detect (known as Low Probability of Detection, or LPD, communications [14]), and there may be some application of chaos in this type of communication. Conventional communications, including spread spectrum, is based on periodic carrier signals, which are obviously artificial and have statistical properties such as cyclostationary [14] that aid in their detection. Chaotic carrier signals may be generated by circuits that simulate natural processes, so they may not be so obviously artificial, and it is possible to generate chaotic carriers which are not cyclostationary. While I am interested in generating LPD signals, I make no attempt in this paper to address questions of security. I am only interested in how difficult it is to determine that a communication signal is present, and not in how difficult it is to extract information from that signal.

II. CYCLOSTATIONARITY

Random signals may have statistical properties that vary periodically with time, in which case they are called cyclostationary [14,15]. Cyclostationarity may be detected in a signal by calculating the autocorrelation of the power spectrum . If $x(t)$ is a signal and $X(f)$ is its Fourier transform, then the autocorrelation of the power spectrum is

$$\Gamma(f) = \frac{\int\limits_{-\infty}^{\infty} X(f) X^*(f-\phi)\, d\phi}{\int\limits_{-\infty}^{\infty} X(\phi)\, X^*(\phi)\, d\phi} \qquad (1)$$

A large cross correlation at a particular frequency in the Fourier spectrum indicates the presence of cyclostationarity (an analogous result holds for discrete signals). From the Wiener-Khintchine theorem [16] , the Fourier transform of the autocorrelation of a signal is the square of the Fourier transform of the signal. To find the autocorrelation of the frequency spectrum, one may simply take the square of $x(t)$, Fourier transform, and search for large components at discrete frequencies.

III. DIRECT SEQUENCE SPREAD SPECTRUM

In order to say something is hard to detect, I must say "hard to detect compared to what". For comparison, I generated a simulated direct sequence spread spectrum (DSSS) signal [17,18]. In direct sequence spread spectrum, the information signal is a digital signal running at some rate n bits/s . The spreading signal is a pseudo-random signal running at a faster rate. For this example, I use a rate of $50n$ bits/s . The spreading signal multiplies the information signal (in binary fashion) to produce the spread information signal, which is then modulated onto a periodic carrier. In order to simulate the modulation, I use a simple modulation called binary phase shift keying (BPSK) [18], where the phase of the carrier is modulated between 2 phases: a phase of 0 for a binary 0 and a phase of π radians for a binary 1.

Since the spreading signal has a greater bandwidth than the information signal, it spreads the spectrum of the periodic carrier. Figure 1 shows the power spectrum of a periodic signal before and after being modulated with a spreading signal. The peak power in the spread signal is greatly reduced from the unspread signal, and that reduction in power is used to hide the spread signal below the background noise level. The DSSS receiver correlates a stored pseudorandom sequence with the transmitted signal to recover the information signal.

IV. CHAOTIC SYSTEM

The direct sequence spread spectrum system is capable of operating at very low signal to noise ratios, but its dependence on a periodic carrier is a weak point. The periodic carrier causes the DSSS signal to be cyclostationary, Even though the signal itself is broadband, squaring the signal makes it much easier to detect.

Although some narrowband chaotic signals may be cyclostationary, it is possible to generate broadband chaotic signals that are not cyclostationary. If the carrier signal is truly chaotic, it never repeats, and a stored reference receiver will not work. Instead, a self-synchronizing chaotic receiver is used. Self-synchronizing systems require more signal energy in noisy environments than stored reference systems, but self-synchronizing systems are potentially simpler to build.

The well known Lorenz system [19] is one example of a broadband chaotic system. The Lorenz system is difficult to build as a circuit, however, and self-synchronized Lorenz systems [4] are sensitive to added noise. I have previously built self-synchronizing chaotic circuits which I could use to communicate when the noise levels were much larger than the signal [20–22], but these circuits required a carrier signal that contained 2 narrowband signals well separated in frequency. In this paper, I design a similar chaotic system that has a broad frequency spectrum centered in one band. At the heart of this broad-band chaotic system is a Rossler-like chaotic system with a variable time constant. This first part of the chaotic system is described by

$$\frac{dx_1}{dt} = -\tau_1\alpha\left(0.05x_1 + 0.5x_2 + x_3\right) \tag{2a}$$

$$\frac{dx_2}{dt} = -\tau_1\alpha\left(-x_1 - 0.15x_2\right) \tag{2b}$$

$$\frac{dx_3}{dt} = -\tau_1\alpha\left(-g\left(x_1\right) + x_3\right) \tag{2c}$$

$$g\left(x\right) = \left\{\begin{array}{ll} 0 & x < 3 \\ 15\left(x-3\right) & x \geq 3 \end{array}\right\} \tag{2d}$$

The system of eq. (2) is just the piecewise-linear Rossler system described in [23] with a variable time constant. The time constant τ_1 is fixed, but the signal α varies with time, effectively varying the time constants of eq.(2), which causes the frequency of the chaotic oscillator to vary. A low frequency limit cycle oscillator generates the time constant variation signal α . The low frequency oscillator is self oscillatory but is also driven by the signal x_1 from eq. (2). The limit cycle oscillator is described by

$$\frac{dx_4}{dt} = -\tau_2 \left(0.02x_4 + 0.5x_5 + x_6 + 0.5\left|x_1\right| + c\xi\right) \tag{3a}$$

$$\frac{dx_5}{dt} = -\tau_2 \left(-x_4 - 0.02x_5 + 2x_3\right) \tag{3b}$$

$$\frac{dx_6}{dt} = -\tau_2 \left(-g\left(x_4\right) + x_6\right) \tag{3c}$$

$$\alpha = 1 + f\left(x_4\right) \tag{3d}$$

$$f\left(x\right) = \left\{ \begin{array}{ll} -0.9 & mx < -0.9 \\ mx & -0.9 \leq mx \leq 0.9 \\ 0.9 & mx > 0.9 \end{array} \right\} \tag{3e}$$

The function f is a bounds function used to keep α from getting too large or too small. The constant m sets the amount of spreading. The signal ζ is a phase synchronization signal (to be described later) used to inject information into the limit cycle oscillator. Typically, the ratio τ_1/τ_2 is approximately 100, so that the signal α causes the time constant for eq. (2) to change slowly in an irregular fashion. Equations (2-3) are enough to produce a broadband chaotic signal, but the phase of the signal x_4 (information will be encoded on the phase of x_4) is easily determined from the envelope of any of the signals from eq. (2). A further step is necessary to reduce the detectability of the information signal. Equation eq. (4) describes a second limit cycle oscillator whose frequency is close to the limit cycle oscillator of eq. (3) but incommensurate

$$\frac{dx_7}{dt} = -\tau_3\beta \left(0.02x_7 + 0.5x_8 + x_9 + 0.5\left|x_1\right| + 0.2x_3\right) \tag{4a}$$

$$\frac{dx_8}{dt} = -\tau_3 \beta \left(-x_7 - 0.02x_2 + 2x_3\right) \tag{4b}$$

$$\frac{dx_9}{dt} = -\tau_3 \beta \left(-g\left(x_7\right) + x_9\right) \tag{4c}$$

$$\beta = 1 + h\left(x_4\right) \tag{4d}$$

$$h\left(x\right) = \left\{ \begin{array}{ll} -0.9 & mx < -0.9 \\ mx & -0.9 \leq mx \leq 0.9 \\ 0.9 & 0.9 < mx \end{array} \right\} \tag{4e}$$

At the same time, the signal α that modulates the time constant for eq. (2) becomes

$$\alpha = 1 + f\left(x_4 x_7\right) \tag{5}$$

and the x_4 equation from eq. (3) becomes

$$\frac{dx_4}{dt} = -\tau_2 \left(0.02x_4 + 0.5x_5 + x_6 + 0.5\left|x_1\right| + 0.2x_7 + c\xi\right) \tag{6}$$

For this paper, $\tau_1 = 10$, $\tau_2 = 0.1$, $\tau_3 = 0.279$, and $m = 0.2$, and eq. (2-6) were numerically simulated with a 4'th order Runge-Kutta routine [16] using a time step of 0.04 s. Figure 2 shows attractors for the chaotic part and the 2 limit cycle oscillators. The chaotic oscillator of eq. (2) is nearly periodic, so the concept of phase may be used to produce a driving signal. The driving signal is the phase of the chaotic Rossler oscillator, and is calculated as

$$z = \frac{x_2}{\sqrt{x_1^2 + x_2^2}} \tag{7}$$

An advantage to using this phase signal to drive the response system is that the peak to peak amplitude is constant, as can be seen in Fig. 3. Inevitably, the amplitude of z will change as a result of the transmission of z, but an automatic gain control may be implemented to restore the amplitude of z at the receiver. The type of receiver used below is also not that sensitive to the exact amplitude of z , so amplitude fluctuations will not have a large effect on the receiver. Figure 3 also shows a power spectrum of z , showing that z is a broadband signal. The shape of the power spectrum of z may be altered by varying the spreading function f in eq. (5).

V. RESPONSE SYSTEM

The chaotic drive system is based on 3 subsystems, all of which oscillate independently, so it is not possible to build a response system which exactly synchronizes to the drive system. The response system is designed so that the limit cycle parts phase synchronize [24] to their counterparts in the drive system, and information is encoded on the phase of the limit cycle. The signal that varies the time constant of the chaotic system may be recovered from the transmitted signal z. Although a short time series of z looks like a periodic signal with a slow frequency modulation, the unmodulated version of z is not periodic but chaotic. The phase of z varies chaotically. Because z is chaotic, the time constant variation can't be recovered by a phase locked loop, which assumes that the carrier phase is constant and all phase variation comes from the modulating signal. Instead, a simpler technique based on a bandpass filter is used. A bandpass filter [25] is modeled by

$$\frac{d\psi_1}{dt} = \frac{1}{R_1 C}\frac{dz}{dt} - \frac{1}{R_2 C}\psi_1 + \psi_2 \tag{8a}$$

$$\frac{d\psi_2}{dt} = -\frac{R_1 + R_3}{R_1 R_2 R_3}\psi_2 \tag{8b}$$

where $R_1 = 14{,}261\ \Omega$, $R_2 = 2R_1$, and $R3 = R1$, and $C = 10^(- 5)$ F. Equation (8) models a bandpass filter with a Q of 1 and a center frequency of 1.117 Hz, corresponding to the peak frequency of the chaotic system in eq. (2) when α is fixed at 1.

The bandpass filter passes signals at the center frequency with no phase shift but shifts the phase of signals not at the center frequency, so the filter output ψ_2 will be shifted in phase from the filter input z by an amount corresponding to the amplitude of the time constant variation signal α . The signal α is recovered approximately as the signal ψ_3, where

$$\frac{d\psi_3}{dt} = \frac{\tau_1}{100}\left(sq\left(z\right)sq\left(\psi_2\right) - \psi_3\right) \tag{9a}$$

$$sq\left(x\right) = \left\{\begin{array}{ll} -1 & x \le 0 \\ 1 & x > 0 \end{array}\right\} \tag{9b}$$

The rest of the response system is described by

$$\gamma = 2 - 0.015\psi_3 \tag{10a}$$

$$\frac{dy_1}{dt} = -\tau_1\gamma\left(0.05y_1 + 0.5y_2 + y_3\right) \tag{10b}$$

$$\frac{dy_2}{dt} = -\tau_1\gamma\left(-y_1 - 0.17z + 0.02y_2\right) \tag{10c}$$

$$\frac{dy_3}{dt} = -\tau_1\gamma\left(-g\left(y_1\right) + y_3\right) \tag{10d}$$

$$\frac{dy_4}{dt} = -\tau_2\left(0.02y_4 + 0.5y_5 + y_6 + 0.5\left|y_1\right|\right) \tag{10e}$$

$$\frac{dy_5}{dt} = -\tau_2\left(-y_4 - 0.02y_5 + 2y_3\right) \tag{10f}$$

$$\frac{dy_6}{dt} = -\tau_2\left(-g\left(y_4\right) + y_6\right) \tag{10g}$$

$$\frac{dy_7}{dt} = -\tau_3\left(0.02y_7 + 0.5y_8 + y_9 + 0.5\left|y_1\right| + 0.2y_3\right) \tag{10h}$$

$$\frac{dy_8}{dt} = -\tau_3\left(-y_7 - 0.02y_2 + 2y_3\right) \tag{10i}$$

$$\frac{dy_9}{dt} = -\tau_3\left(-g\left(y_7\right) + y_9\right) \tag{10j}$$

$$g\left(y\right) = \left\{\begin{array}{ll} 0 & y < 3 \\ 15\left(y-3\right) & y \geq 3 \end{array}\right\} \tag{10k}$$

The response system is not an exact replica of the drive system, and synchronization is not exact. There is phase synchronization between y_4 in the response and x_4 in the drive (as shown in Fig. 4), so information is transmitted by modulating the phase of x_4 [26]. The phase modulation signal ξ in eq. (3) is set equal to

$$\xi = s_i \sin(\omega_1 t) - 0.1x_4 \tag{11}$$

where $s_i = \pm 1$ depending on the value of the binary information signal, and the coupling constant $c = 0.2$, and $\omega/2\pi = 0.0115$ Hz, the frequency of the limit cycle oscillator of eq. (3). The phase of y_4 is determined by a phase locked loop [25] in the receiver.

VI. INFORMATION DETECTION

There are 2 types of detection considered in this paper, detection of the information contained in the communication signal and detection of the communications signal itself. Producing a signal which can't be detected by an adversary is not very useful if we can't detect the information content ourselves, so detection of the information content is considered first. As described above, the information signal is phase-modulated onto the x_4 signal in eq. (3) and detected by detecting the phase of the y_4 signal in eq. (10). The standard method that engineers use to characterize communication efficiency is to plot the bit error rate ($B_(ER)$) [18] as a function of the energy in one bit, normalized by the noise power spectral density (a flat noise spectrum is assumed), abbreviated as E_b/N_0.

Figure 5 is a plot of the performance for the transmitter and receiver of eq. (2-11), with noise that occupies the same bandwidth as the signal. Fig. 5 shows performance for the receiver of eq. (8-10) and a receiver where the phase is detected directly from the recovered time constant modulation signal ψ_3 in eq. (9). The performance plot for ψ_3 is included to show that it is more difficult to detect the information content of the communication signal if one doesn't know specific receiver details. The bit energies required to achieve low bit error rates in Fig. 5 are actually quite large compared to the bit energies required for conventional communications signals [18] (such as the DSSS signal), but the goal in those cases is to make signals that are easy to detect, while the goal in this paper is to create s signal that is hard to detect. The increased bit energy required to detect the information content for the present method is due to 2 things: self synchronizing receivers do not perform as well as the stored reference receivers used in conventional communications, and the y_4 signal in the response system takes a long time to phase synchronize with the x_4 signal in the drive system. The coupling between the limit cycle oscillator that generates x_4 (and y_4) and the chaotic system that generates the transmitted signal is weak and nonlinear, so synchronization is slow. The weak coupling is necessary to make the communications signal harder to detect. In practical applications, there may be hardware considerations that make the higher power requirements of the current method less of a disadvantage.

VII. DETECTABILITY

For covert communications, it is important that even the presence of a communications signal not be detectable, since the signal is a beacon that gives away the location of the transmitter. The simplest way to detect a signal is to detect the signal power, but if there are other features of the signal, such as cyclostationarity, then signal detection can be easier than simply looking for power.

In order to calculate the probability of detection [14], some signal statistic is chosen, and the probability distribution for this statistic is found when there is no signal (only noise is present) and when the signal plus noise are present. The DSSS signal is cyclostationary [15], so this property is used to aid in detection. In the power spectrum of the squared DSSS signal in Fig. 6, a peak at twice the carrier frequency is obvious. I estimate the probability distribution of the power at this frequency with noise only or with noise plus signal present. The overlap in these probability distributions is the probability of either falsely detecting a signal or missing the presence of a signal. Subtracting the overlap area from 1.0 gives the probability of detection. Figure 7 shows the probability of detection P_d as a function of signal to noise ratio S_N for the DSSS signal.

Figure 8 is the power spectrum of the squared signal z, the transmitted signal from the chaotic system. The power spectrum of z^2 is still broad band because there are no strong correlations in its frequency spectrum as there were for the DSSS signal with a periodic carrier. The presence of the signal z can only be inferred by looking for transmitted power, a method known as the radiometer method [14], so the chaotic signal will be harder to detect. Probability distributions for noise only and noise plus the signal z are estimated by measuring the average power present in the signal. The resulting probability of detection is also plotted in Fig. 7 as a function of signal to noise ratios. Eventually the curves for the DSSS signal and the chaos signal in Fig. 7 come together because in both cases the probability of detection approaches 0 for low signal to noise ratios.

VIII. CONCLUSIONS

This paper has demonstrated that the lack of a periodic carrier signal makes chaotic signals easier to hide than conventional digital communications signals. The amount of energy needed to transmit this particular chaotic signal was large, but the main point here was to demonstrate that a noncyclostationary chaotic signal could be used as a carrier signal, even when background noise was larger than the signal. There are other properties of communications signals that one may also want to hide. Higher order statistics can also reveal the type of modulation used, [14] which can give information about who sent the signal. It is also possible to detect the chip rate [18] in a communications signal, which is the clock rate for the underlying digital system. It may be possible to better conceal this identifying information by using nonperiodic chaotic signals to carry the information signal.

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FIGURES

FIG. 1. (a) Power spectrum of a sinusoidal carrier signal before modulation with a direct sequence spread spectrum signal. (b) Power spectrum of the carrier signal after modulation with a direct sequence spread spectrum signal.

FIG. 2. (a) Attractor from chaotic part of the driving system eq. (2). (b) Attractor from first limit cycle oscillator of the driving system of eq. (3). (c) Attractor from second limit cycle oscillator eq. (4) from the driving system.

FIG. 3. (a) Transmitted signal z. (b) Power spectrum of transmitted signal z.

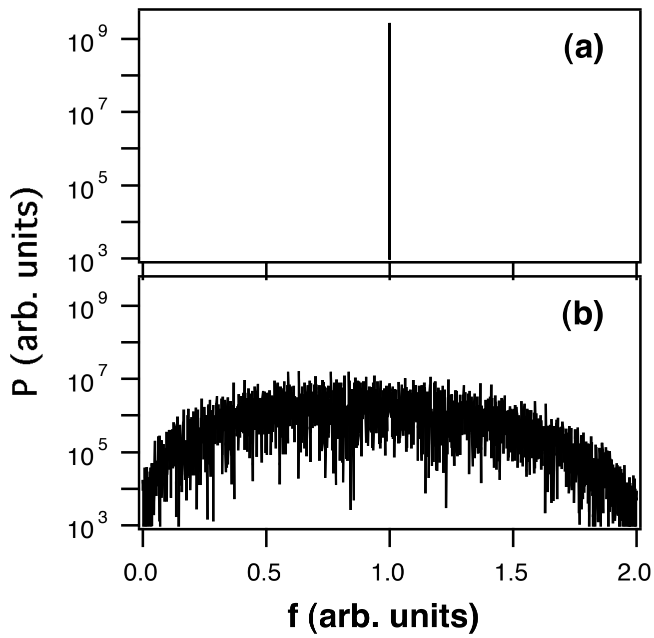
FIG. 4. Signal y_4 from the response system (eq. 10) plotted vs. x_4 from the driving system of eq. (3), showing phase synchronization.

FIG. 5. Bit error rates $(B_(ER))$ as a function of energy per bit normalized by noise power spectral density (E_b/N_0) for two different receivers. The black circles are for the full chaotic response system of eq. (8-10), while the open squares are for a conventional receiver defined by eqs. (8-9).

FIG. 6. Power spectrum of the squared direct sequence spread spectrum signal.

FIG. 7. Probability of detection P_d as a function of signal to noise ratio S_N for the direct sequence spread spectrum signal (open squares) and the chaotic signal (black circles).

FIG. 8. Power spectrum of z^2 , where z is the chaotic communication signal of eq. eq. (7).



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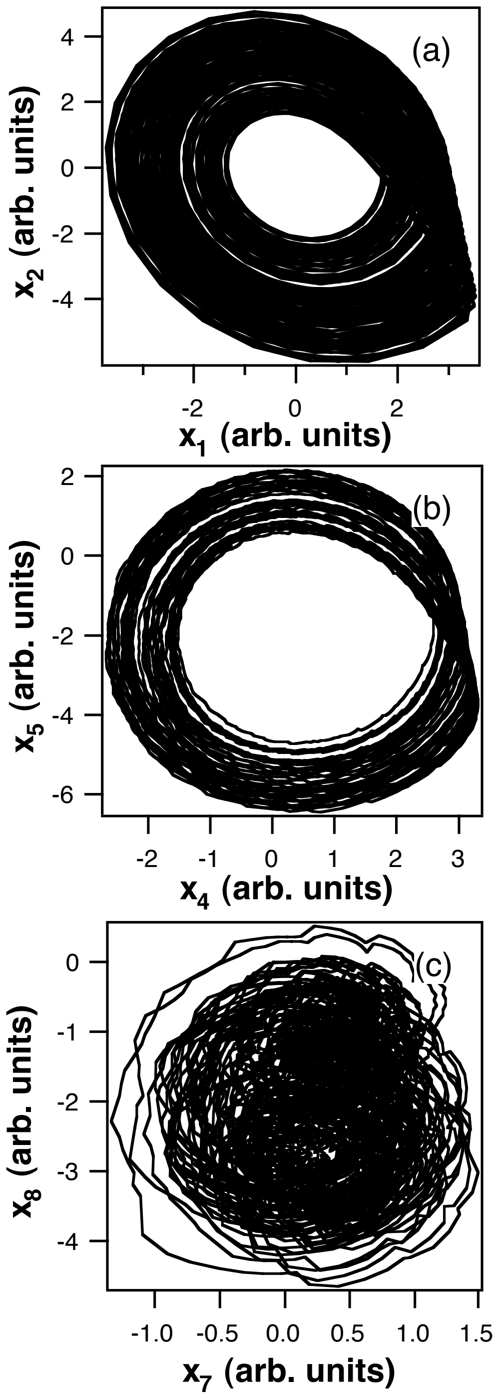

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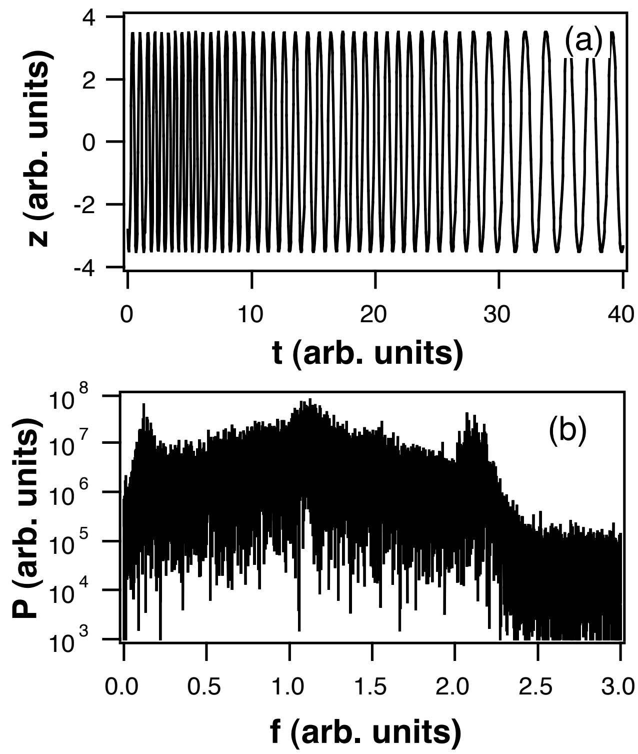

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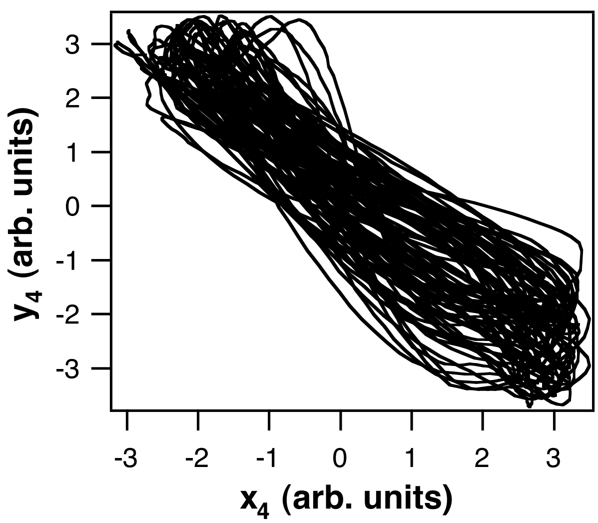


FIG. 4. Signal y4 from the response system (eq. 10) plotted vs. x4 from the driving system of eq. (3), showing phase synchronization.

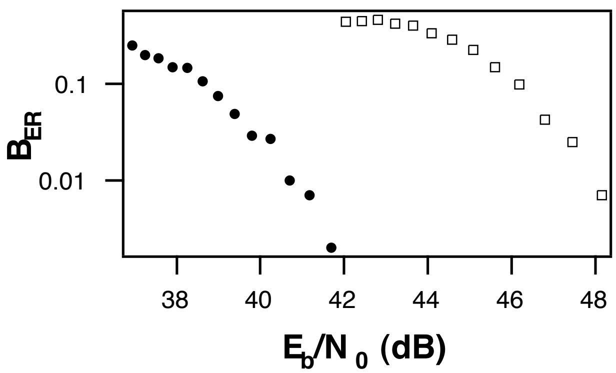
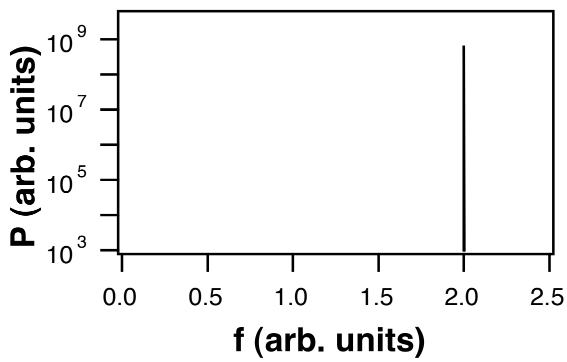


FIG. 5. Bit error rates (B (ER)) as a function of energy per bit normalized by noise power spectral density (Eb =N0) fortwo di□ erent receivers. The black circles are for the full chaotic response system of eq. (8-10), while the open squares are for a conventional receiver de□ ned by eqs. (8-9).



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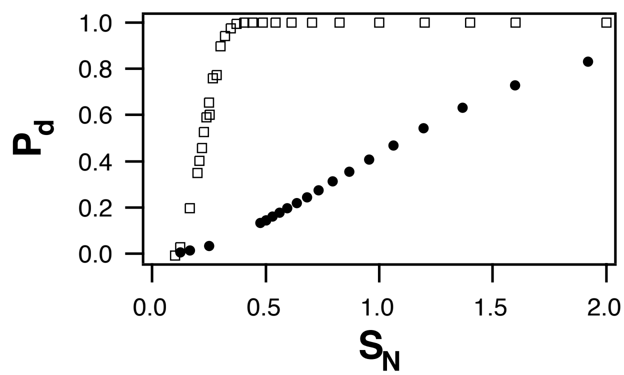

FIG. 7. Probability of detection Pd as a function of signal to noise ratio SN for the direct sequence spread spectrum signal (open squares) and the chaotic signal (black circles).

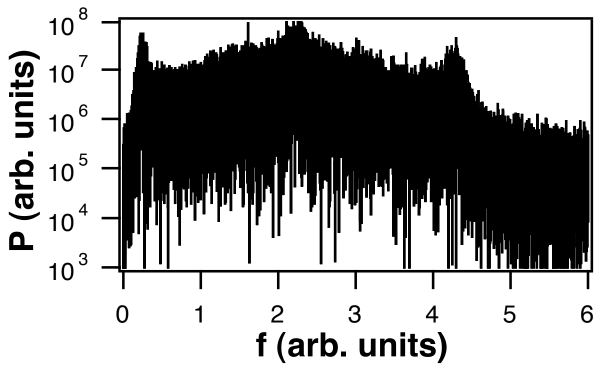

FIG. 8. Power spectrum of z 2 , where z is the chaotic communication signal of eq. eq. (7).