Using positive entropy signals to increase communications efficiency

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***Abstract*—Conventional communication signals may be separated into a low frequency baseband signal, which contains all the information, and a higher frequency periodic carrier signal. I show here that one may achieve communications signals with higher capacities by generating signals that can not be separated into carrier and baseband signals. The signals I consider have higher entropy than signals that are separable into carrier and baseband, and entropy is an upper limit on information capacity. To demonstrate these ideas, I develop a communications system inspired by CW radar; several delayed versions of a known signal are added together, with the set of delay values defining a symbol. The same undelayed signal is also known by the receiver (this is a stored reference method), making it possible to extract the delay values. Simulations show that high bandwidth efficiencies are possible with this system.**

***Index Terms*—bandwidth efficient, chaos, communication, noise**

I. Introduction

The reason for studying the application of signals such as chaos to communications is that chaotic signals have a positive entropy, and the information capacity of a signal is limited by its entropy [1]. Hayes et al. [2] suggested that chaotic carriers should make excellent information carriers because of this positive entropy, and several groups have attempted to apply some of Hayes' ideas [3-6]. Other signals with positive entropy, such as noise, may also be useful information carriers. I consider chaotic signals in this paper because in some cases it is possible to calculate their entropy.

In this work, I will introduce an efficient communications method that depends on the properties of signals with positive entropy. Typically in communications, a baseband signal is modulated onto a periodic carrier signal. Because the carrier signal is periodic it has 0 entropy and contains no information. One finds the information capacity of the signal by considering only the baseband signal.

One may instead generate a communications signal that can't be separated into baseband and carrier signals. One way to do this is to bandpass filter a chaos or noise signal; one may also multiply a high frequency narrow band chaos (or noise) signal by a lower frequency chaos (or noise) signal. The absence of a periodic carrier makes it possible for this type of signal to have a higher entropy, and therefore a higher information capacity.

Alternatively, the advantage of a signal that can't be separated into baseband and carrier signal may be explained by using the cross correlation properties of the signal. I consider two reference signals, both of which are Gaussian white noise signals, both filtered to have a bandwidth of 0.25 Hz. A third filtered Gaussian noise signal matches one of the two reference signals, but it has filtered Gaussian noise added to it. The cross correlations $\rho_{xy}(t)$ between the two reference signals and the noisy signal may be calculated in order to determine which of the two reference signals matches the noisy signal. The cross correlations have a certain variance, which may be estimated from a formula due to Bartlett [7]. In the long time limit, the Gaussian signals have zero cross correlation, and the approximation for the variance reduces to $\mathrm{var}\left[r_{xy}(k)\right] = \left\{\sum_{v=-\infty}^{\infty} \rho_{xx}^{2}(v)\right\} / (n-k)$, where ρ_{xx} is the autocorrelation, k is the lag time, and n is the number of points used to calculate the cross correlation.

The auto correlation function of a lowpass filter is known to be $\sin(\omega_R t)/(2\pi t)$ [8]. A bandpass filter may be written as the difference between two lowpass filters, so the autocorrelation of a bandpass filtered Gaussian signal is $[\sin(\omega_1 t) - \sin(\omega_2 t)]/(2\pi t)$, where ω_1 is the upper end of the pass band, and ω_2 is the lower end.

Applying the variance approximation above, the variance of the cross correlation estimate between Gaussian signals that are filtered between 0 and 0.25 Hz is 0.05/T, where T is the time over which the cross correlation is calculated. The variance of the cross correlation estimate between Gaussian signals that are filtered between 1 and 1.25 Hz is 0.0025/T. Although all the Gaussian signals have the same bandwidth, the cross correlation estimate for the higher frequency signal is more accurate because its variance is smaller. If the baseband signal (0 to 0.25 Hz) was simply modulated onto a carrier to shift its frequency range up to 1 to 1.25 Hz, the variance in the cross correlation estimate would still be 0.05/T, so the signal that can't be broken into a baseband signal and a carrier signal gives a better estimate of the cross correlation.

II. Delay Communication Method

Knowing that a signal has a positive entropy is not always

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enough to use the full information carrying capacity of that signal. One must also find a way to make use of the entropy. The delay communication method described here depends on the positive entropy of the communications signal to make the detection of delays possible.

Begin with a noise or chaos signal, $x(t) = x(t_1, t_2, t_3 \ldots)$. Both transmitter and receiver have a copy of this signal. A symbol is encoded by choosing a set of delay values, $(\tau_1, \tau_2, \tau_3 \ldots)$. The signal transmited is then

$$\xi_t(t) = x(t+\tau_1) + x(t+\tau_2) + x(t+\tau_3) + \cdots \qquad (1)$$

$\xi_t(t)$ is transmitted continuously for some time interval L . At the end of the interval, a new symbol is encoded by choosing a new set of delay values. I continue to transmit $\xi_t(t)$ continuously, but it changes in each interval because the set of delay values changes. For now the effects of a channel with finite bandwidth are not considered; it will be mentioned later how finite bandwidth affects the technique.

The receiver also has a copy of $x(t)$ (this is a stored reference method) and knows the interval length L. At the receiver, a reference signal is formed:

$$\xi_r(t) = x(t+\tau_i) + x(t+\tau_j) + x(t+\tau_k) + \cdots \qquad (2)$$

The cross correlation between $\xi_t(t)$ and $\xi_r(t)$ is found for all possible combinations of delays in $\xi_r(t)$ in an attempt to identify the delay values used to create $\xi_t(t)$. When the set of delays in the receiver $(\tau_i, \tau_j, \tau_k \ldots)$ is equal to the set of delays in the transmitter $(\tau_1, \tau_2, \tau_3 \ldots)$, then the cross correlation between $\xi_r(t)$ and $\xi_t(t)$ will be a maximum.

To create the transmitted signal, n delayed versions of the signal $x(t)$ are used. Each of the n delays may take on m values; for example, for $n = 3$ and $m = 5$, 3 delays must be chosen from a set such as (10s, 20s, 30s, 40s, 50s). Delay combinations that might chosen could be (10s, 20s, 30s), or (10s, 30s, 50s), or other such combinations. At the receiver, these delay values can be detected, but their ordering can't be, so the order of these sets is not important. Multiple occurrences of the same delay in any set are not allowed. Under these rules, the number of symbols (delay combinations) possible for a set of n delays each with m possible values is

$$N_s = \frac{m(m-1)(m-2)\ldots(m-n+1)}{n!} \qquad (3)$$

The combinatorial nature of this method means that it is possible to create a large number of symbols from a set of delays.

III. Demonstration with Chaotic Maps

The delay communication method is first demonstrated with 1-dimensional chaotic maps. These maps are used so that I may demonstrate the connection between increased entropy and increased capacity for this method. In general, calculating entropy for a signal is difficult because one must know the generating partition for the signal, but for 1-dimensional maps, the generating partition is defined by the set of critical points for the map.

The entropy of the map signal is an upper bound for the capacity C of the map signal, and the capacity tells us the minimum signal to noise ratio for which error free communication is possible:

$$C = \frac{1}{2}\log\left(1 + \frac{S}{N}\right) \qquad (4)$$

where S/N means signal power divided by noise power.

If the generating partition for a signal is known, the entropy can be calculated from symbolic dynamics[5, 9]. Each part of the partition forms a symbol; for example, for the logistic map shown below in eq. (5), the regions of the partition are A: $0 < y_n \le (1/2.1)$; B: $(1/2.1) < y_n \le (2/2/1)$; C : $(2/2.1) < y_n < 1$. When < 1. When y_n falls into region A, symbol A is produced; in region B, symbol B, etc. The entropy may then be calculated from the discrete definition, $H = -\sum_{i=1}^{N_S} p(s_i)\log[p(s_i)]$ [10], where $p(s_i)$ is the probability of symbol s_i and N_s is the number of symbols.

A shift map is first used to simulate a low frequency signal. The map is iterated once every 20 steps in order to keep the frequencies low:

$$y_{n+20} = 2.1 y_n \mod 1 \qquad (5)$$

The power spectrum of the output from this map has a bandwidth of approximately 0.05 Hz.

The logistic map, $z_{n+1} = \mu z_n(1 - z_n)$ was used to generate a higher frequency signal. The logistic map signal was used in a fashion similar to a carrier signal, in that it was used to raise the frequency of the shift map signal, but the logistic map signal was not a carrier. A carrier signal adds no information to the transmitted signal- it is simply used to shift up the frequency of the information signal prior to transmission. The logistic map signal in these simulations was part of the information signal. As a result, the signal that was transmitted could not be divided into a carrier signal and a baseband signal. The logistic map was used with 2 different values for the parameter μ : for $\mu = 3.5$, z_n was a periodic signal (period 4); for $\mu = 3.6$, z_n was chaotic, with an entropy of 0.94 bits. This entropy number does not actually indicate how many bits may be transmitted by this method- it is just an upper bound for what may be transmitted using this signa

The reference signal was then the product of the shift map and logistic map signals: $x_n = y_n z_n$. Four delays were used with x_n, each with 10 possible values, for a total of 210 possible symbols, or 7.7 bits for each interval. At 1s per iteration, each interval L was 100s long. The possible delay values were spaced every 100 s, i. e. the set of possible delay values was (100s, 200s, 300s . . .). Gaussian white noise was added to x_n in order to find a bit error rate.

Figure 1 shows the bit error rate as a function of S/N. It can be seen that multiplying the shift map signal by the positive entropy chaotic signal results in a lower bit error rate than multiplying by the 0 entropy periodic signal. In hindsight, it seems obvious that the product of the chaotic logistic map and the shift map has more variation, so it should be easier to distinguish different delay combinations than it should be for the product of the shift map and periodic signal, which has less variation. Entropy is a way of quantifying the amount of variation.

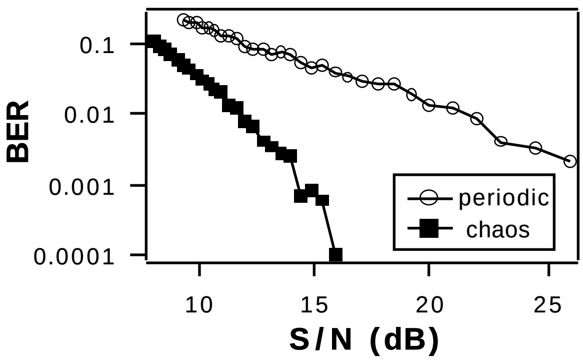

Figure 1. Bit error rate (BER) for a chaotic signal from the shift map when it is multiplied by a periodic or a chaotic signal from the logistic map. When the chaotic signal is used, the bit error rate is lower.

IV. Flows

The maps described above were useful for illustrating the general concepts of the delay communication method, but they did not have high bandwidth efficiencies. Higher bandwidth efficiencies were seen in chaotic flows.

The chaotic system used was designed to have a well defined and controllable bandwidth. The system is a 6-dimensional version of a Rossler-like system described by

$$\frac{dx_1}{dt} = -\alpha\left(0.02x_1 + 0.5x_2 + x_3 + 0.5|x_4|\right)$$

$$\frac{dx_2}{dt} = -\alpha\left(-x_1 + 0.02x_2 + x_6\right) \qquad (6)$$

$$\frac{dx_3}{dt} = -\alpha\left(-g(x_1) + x_3\right)$$

$$\frac{dx_4}{dt} = -10\alpha\beta_f\left(0.05x_4 + 0.5x_5 + x_6\right)$$

$$\frac{dx_5}{dt} = -10\alpha\beta_f\left(-x_4 - 0.13x_5\right) \qquad (7)$$

$$\frac{dx_6}{dt} = -10\alpha\beta_f\left(-g(x_4) + x_6\right)$$

$$g(x) = \begin{cases} 0 & x < 3 \\ 15(x-3) & x \geq 3 \end{cases} \qquad (8)$$

$$\beta_f = 1 + \gamma\left(x_2 + 1.75\right)$$

where α, which sets the overall time scale of the oscillator, is 1 [11] . Equation (7) is a Rossler oscillator [12] whose time constant is modulated by β_f , which is a function of x_2 from eq. (6). Equation 6 is a low frequency nonlinear oscillator which is driven by the higher frequency Rossler system of eq. (7). The effect of the variable time constant β_f is to broaden the power spectrum of the Rossler system of eq. (7) by an amount determined by the factor γ in eq. (8). For the simulations presented here, $\gamma = 0.3$. Equations (6-8) were numerically integrated with a 4'th order Runge-Kutta integration routine with a time step of 0.04.

The transmitted signal was derived from

$$x_t(t) = \frac{x_5(t)}{\sqrt{x_4(t)^2 + x_5(t)^2}} \qquad (9)$$

which is the sine of the phase angle of the Rossler attractor. The signal $x(t)$ has a constant envelope. The signal $x_t(t)$ has a bandwidth of 0.25 Hz . The delay communication signal $\xi_t(t)$ (defined in eq. (1)) was also bandpass filtered by a digital FIR filter with a bandwidth of 0.25 Hz to insure a well defined bandwidth.

In order to maximize the bandwidth efficiency of the delay communication method with the Rossler signal, I also used a channel width of 0.25 Hz. As a result, there was intersymbol interference from one data interval to the next, because the finite bandwidth meant that the transmitted signal $\xi_t(t)$ could not change instantaneously to reflect a new symbol. As a result, an equalization scheme was necessary. The filters used here were linear FIR filters, so the reference signal in the receiver was prefiltered to take the finite bandwidth into account. The finite bandwidth also caused intersymbol interference, so once a symbol was detected, its effect on the next symbol was calculated and used to improve the estimate of the next symbol.

The Rossler signal was transmitted with 4 delays. The possible delays ranged from 0.4 s (10 points) to 40 s (1000 points), and were spaced every 4 s (100 points), for 10 possible delay values. Each data interval was 4 s long. The transmitted delays were detected by computing the cross correlation of a stored reference signal with the transmitted signal. Applying eq. (3), 4 delays with 10 possible values gave a total of 210 possible symbols, or 7.7 bits (the logarithm base 2 of 210). The bandwidth efficiency of this method was 7.7 bits/4 s/0.25 Hz, or 7.7 bits/s/Hz.

Figure 2 shows the simulated bit error rate as a function of E_b/N_0 for this method. I used 10,000 data intervals for this simulation. The noise signal was also filtered to the channel width of 0.25 Hz. The receiver did not actually detect individual bits, but detected the set of delay values (τ_1, τ_2, τ_3, τ_4). Noise could result in an error in any one of these detected delay values. There were 10×9×8×7/4! = 210 possible combinations of these delays, for 7.7 bits. If one delay was in error, there were 10×9×8/4! = 30 possible combinations one could make from only 3 correct delays, so there were 210-30 = 180 fewer combinations than with no error. One way of stating this is that $\log_2(180) = 7.5$ bits were lost. This is almost all 7.7 bits, so I round up to say that all 7.7 bits were lost. Then the bit error rate is #errors/(total bits) = #errors×7.7/(#intervals×7.7) = #errors/10,000. The quantity E_b/N_0 = [(signal power)×(interval length)/(bits per interval)]/[(noise power)/(channel bandwidth)].

On fig 2 I have also plotted the theoretical E_b/N_0 for 128 level QAM, calculated from the formula in [13]. 128 level QAM has a bandwidth efficiency of 7 bits/sec/Hz ($\log_2(128)$ = 7). The chaotic signal with slightly better bandwidth efficiency has a slightly better bit error rate than 128 QAM.

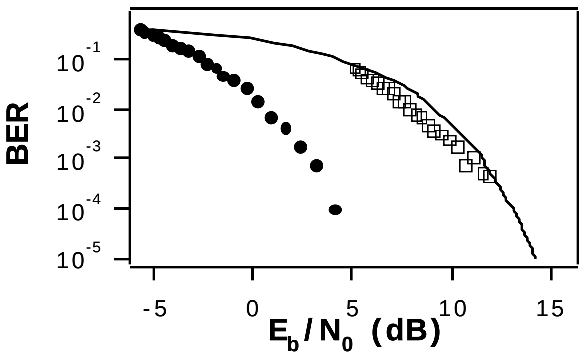

Figure 2. The squares are the bit error rate for delay communication with the flow of eqs. (6-8) using 4 delays with 10 delay values, an interval length of 4 s, and a bandwidth of 0.25 Hz, for a bandwidth efficiency of 7.7 bits/sec/Hz. The dots are for the same parameters, but for a filtered noise signal (same bandwidth efficiency). The solid line is the theoretical bit error rate for 128QAM, which has a theoretical bandwidth efficiency of 7 bits/sec/Hz.

Since higher entropy signals have higher capacities, it is useful to use this delay method with a noise signal, which has infinite entropy. The capacity of the noise signal is limited only by the channel bandwidth[4]. A computer generated Gaussian white noise signal was bandpass filtered by an FIR filter to a bandwidth of 0.25 Hz, and used as a reference signal for the delay communication method, with 4 delays, 10 possible delay values, and a data interval of 4s. Figure 2 also shows the bit error rate for this noise signal, which also had a bandwidth efficiency of 7.7 bits/sec/Hz. Clearly, the highest entropy signal gives the lowest bit error rate. Deterministic chaotic signals might still be useful if one wanted to insure that different transmitters generated different signals, since one can design chaotic systems to be different.

V. CONCLUSION

Signals that can't be separated into carrier and baseband appear to be capable of having a larger entropy than signals that can be separated, and therefore they can have the capacity to carry more information. One reason for this is that the periodic carrier does not increase the entropy of the signal.

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