Using Nonlinear Dynamics to Improve Chaotic Synchronization

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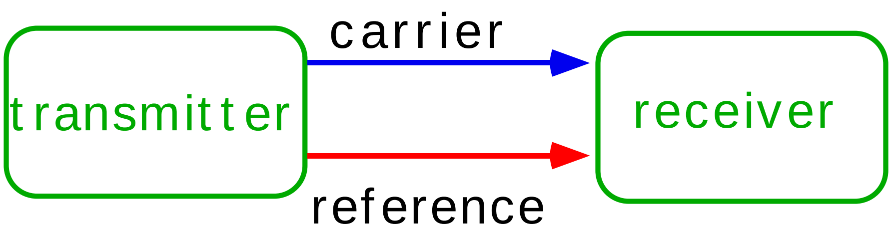
Is chaos useful for communications?

Self-synchronizing systems:

• "Transmitted reference" systems developed in 1960's.



Added noise contaminates reference. Easy to jam because 2 signals

• "Self-synchronizing" chaotic systems

Same noise problems.

Only 1 signal required.

• Non-synchronizing chaotic systems.

Similar to CDMA

If “transmitted reference” methods already known, why study chaotic self-synchronization?

Chaotic systems are dynamical systems. We may be able to manipulate them in new ways.

Look at physics of coupled dynamical systems

How to get receiver to synchronize most closely to drive in the presence of:

- **additive noise**
- **phase distortion**

Main concern: get signal from A to B considering only these simplest problems. Worry about engineering measurements later.

Build up a set of tools for improving synchronization. <u>Don't</u> try to solve entire problem at once.

Methods used:

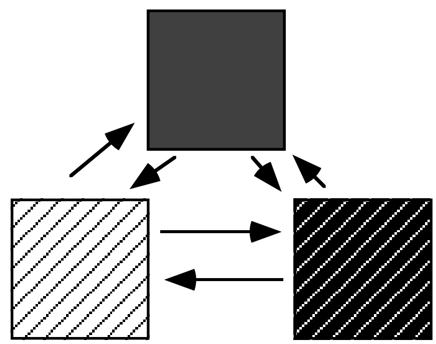
- **choice of dynamical system (2-attractor systems)**

- **improve receiver stability through:**
  - **synchronous substitution**
  - **generalized coupling with minimization**

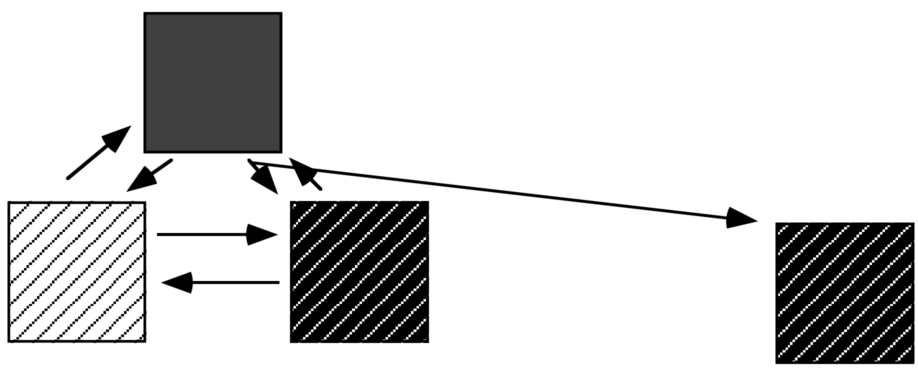
- **vary transmitted signal properties**
  - **synchronous substitution**
  - **coupling through functions:**
    - **- linear filters**

- **"Stored reference" methods**
  - **similar to CDMA**

Synchronizing Chaotic Systems:



find subsystems of chaotic system



drive a subsystem

if all Lyapunov exponents of driven subsystem < 0,
will synchronize

Synchronizing chaotic systems

$$\frac{dx}{dt} = f(x,y,z)$$

$$\frac{dy}{dt} = g(x,y,z) \qquad \frac{dy'}{dt} = g(x,y',z')$$

$$\frac{dz}{dt} = w(x,y,z) \qquad \frac{dz'}{dt} = w(x,y',z')$$

drive system

response system

$$y'(t) \rightarrow y(t)$$

$$z'(t) \rightarrow z(t)$$

if: all Lyapunov exponents for response system are < 0

Cascading synchronized systems

$$\frac{dx}{dt} = f(x, y, z)$$

$$\frac{dy}{dt} = g(x, y, z)$$

$$\frac{dz}{dt} = w(x, y, z)$$

drive system

$$\frac{dy'}{dt} = g(x, y', z')$$

$$\frac{dz'}{dt} = w(x, y', z')$$

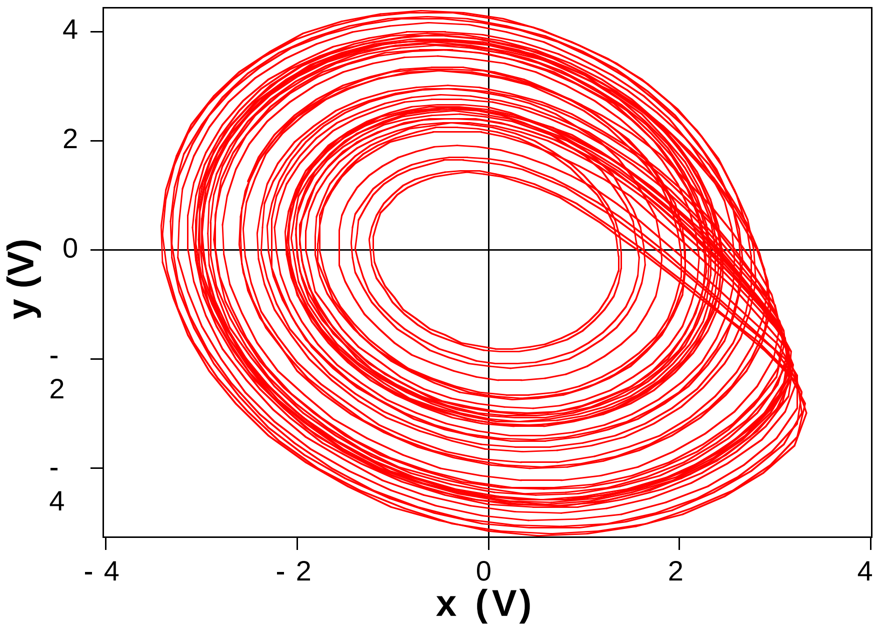
$$\frac{dx''}{dt} = f(x'', y', z'')$$

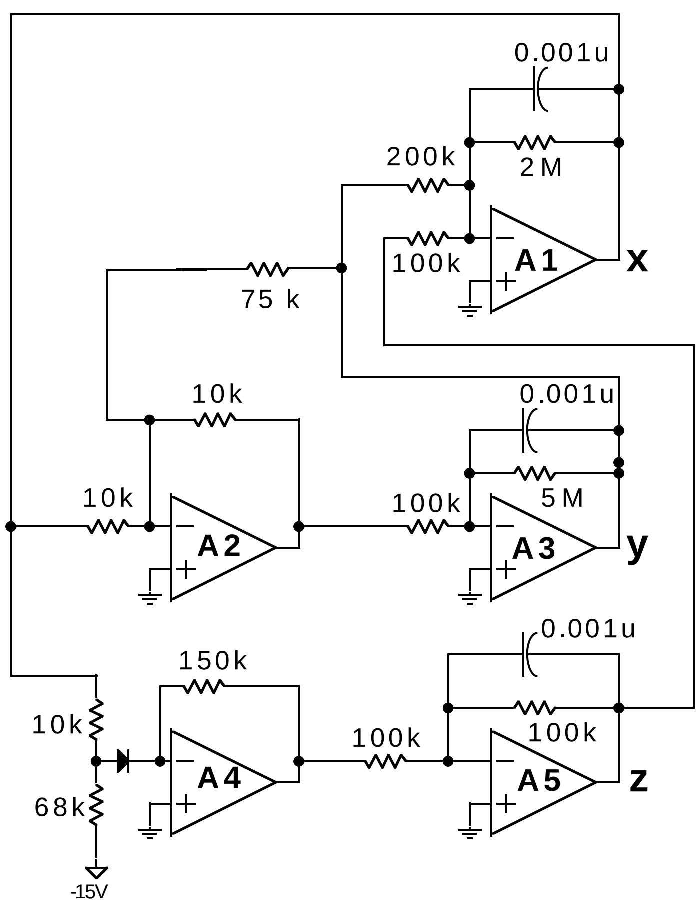
$$\frac{dz''}{dt} = w(x'', y', z'')$$

response system

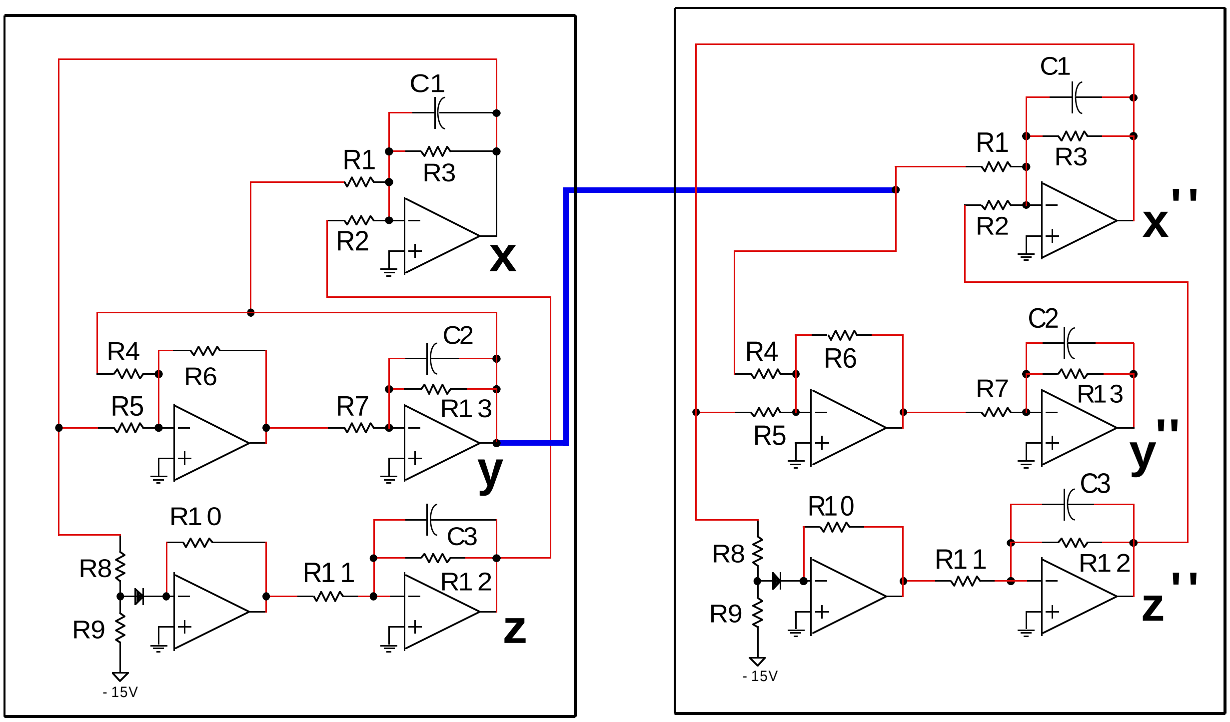
$$x''(t) \rightarrow x(t)$$

Rossler-like circuit





PL Rossler circuit synchronization



Drive **Response**

Example: Synchronizing PL Rossler:

Drive Jacobian

$$\begin{bmatrix} -0.05 & -0.5 & -1.0 \\ 1.0 & 0.131 & 0 \\ \frac{\partial g(x)}{\partial x} & 0 & -1 \end{bmatrix}$$

Response Jacobian

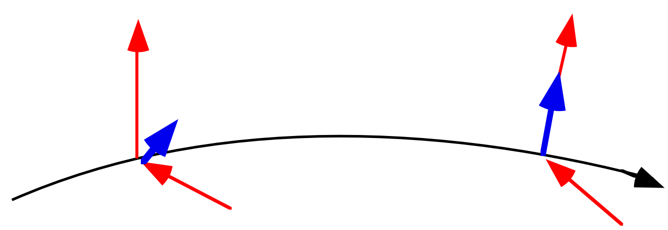
$$\begin{bmatrix} -0.05 & -0.5 & -1.0 \\ 1.0 & 0-0.02 & 0 \\ \frac{\partial g(x)}{\partial x} & 0 & -1 \end{bmatrix}$$

Simple Lyapunov Exponent Calculation
(Largest exponent only)

choose random unit vector ξ

$$\begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} \frac{d\xi_1}{dt} \\ \frac{d\xi_2}{dt} \\ \frac{d\xi_3}{dt} \end{bmatrix}$$

ξ will line up with unstable direction



Simple Lyapunov Exponent Calculation

Occasionally renormalize ξ(t)

$$\ln\left(\frac{\xi(t)}{\xi(0)}\right) \to \lambda_{\max}$$

For piecewise linear systems: can just take eigenvalues of Jacobian.

Flow	**(linear)map**
> 0 : expansion	**abs > 1: expansion**
< 0: contraction	**abs < 1: contraction**

Complications in Stability Calculations

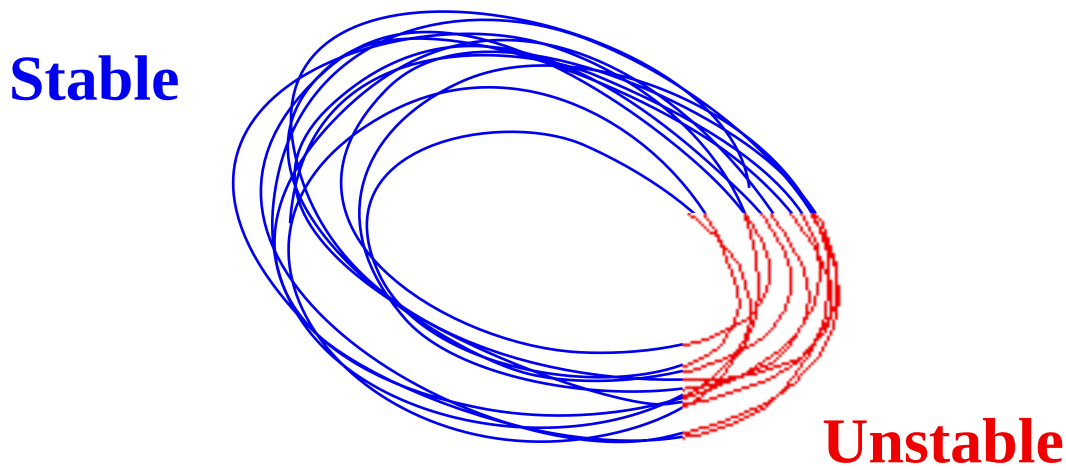
Many different stability criteria for identical synchronization.

Typical problems:

- **Local instabilities**
- **Non-normal eigenvalues**

Local instabilities

Response system:



Transverse Lyapunov Exponents (for coupled systems)

Example: 2 3-dimensional systems

$$\dot{x} = F(x) \quad \dot{y} = F(y)$$

New coordinates:

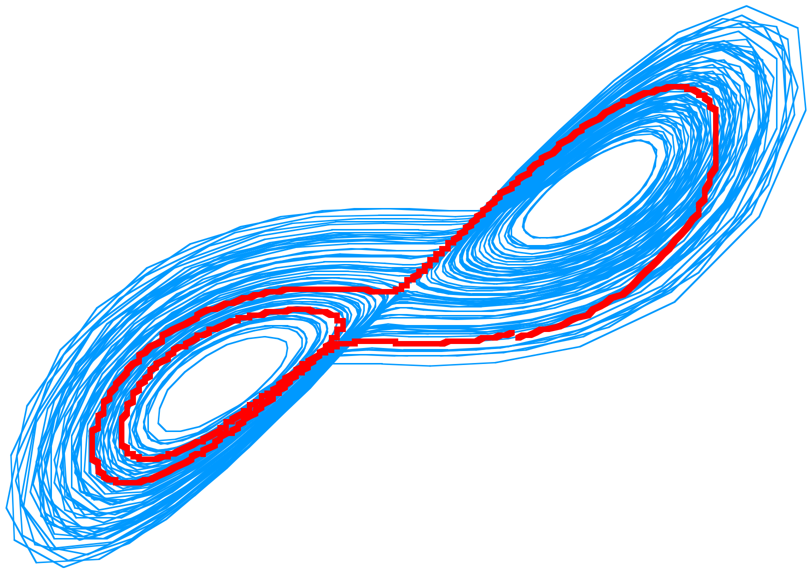
$\xi_1 = x_1+y_1$ $\quad$ $\xi_4 = x_1-y_1$

$\xi_2 = x_2+y_2$ $\quad$ $\xi_5 = x_2-y_2$

$\xi_3 = x_3+y_3$ $\quad$ $\xi_6 = x_3-y_3$

ξ 1, 2 and 3 are motion on the synchronization manifold.
4,5 and 6 are motion transverse to the synchronization manifold

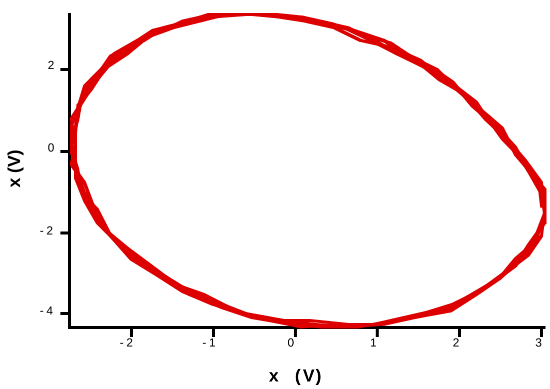
Unstable periodic orbits



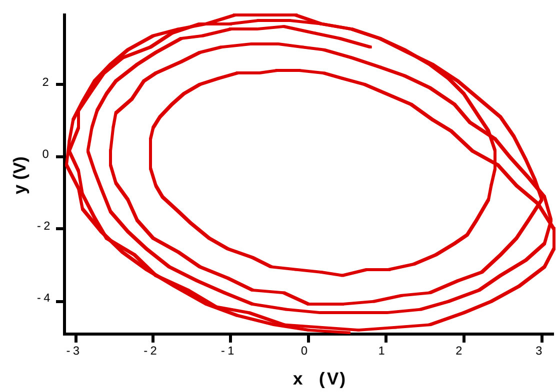
A chaotic attractor consists of an infinite number of unstable periodic orbits (UPO's)

PLR circuit

period 1

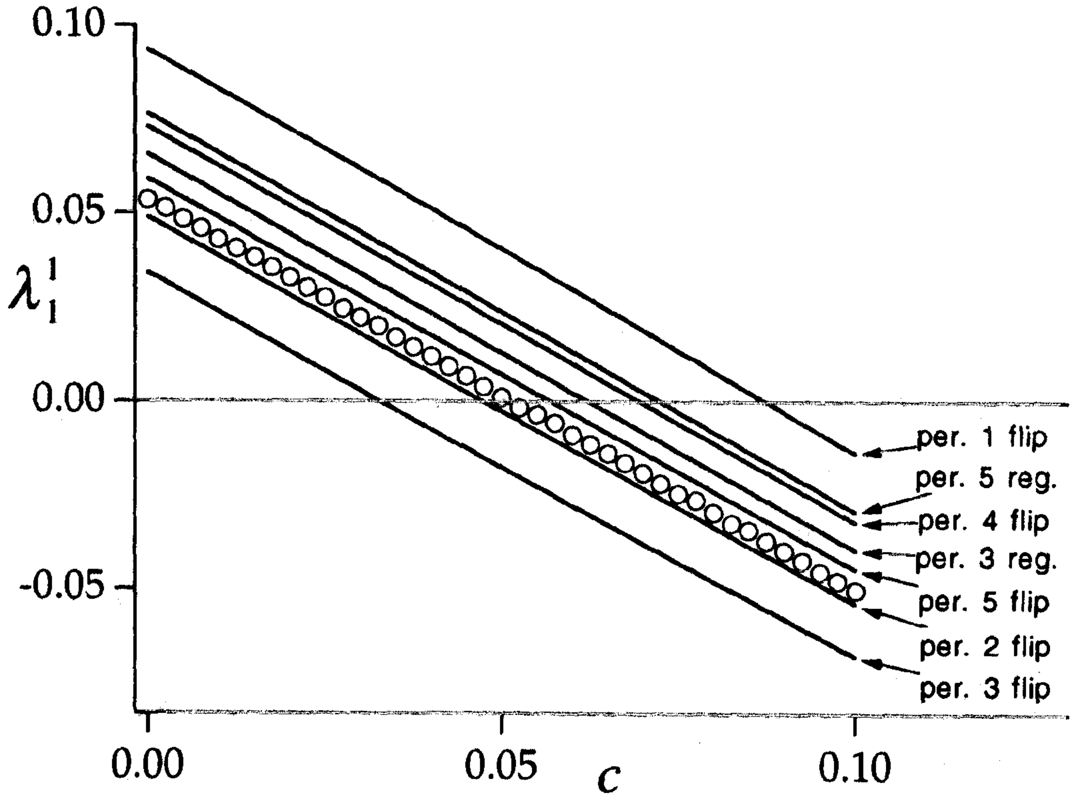


period 4

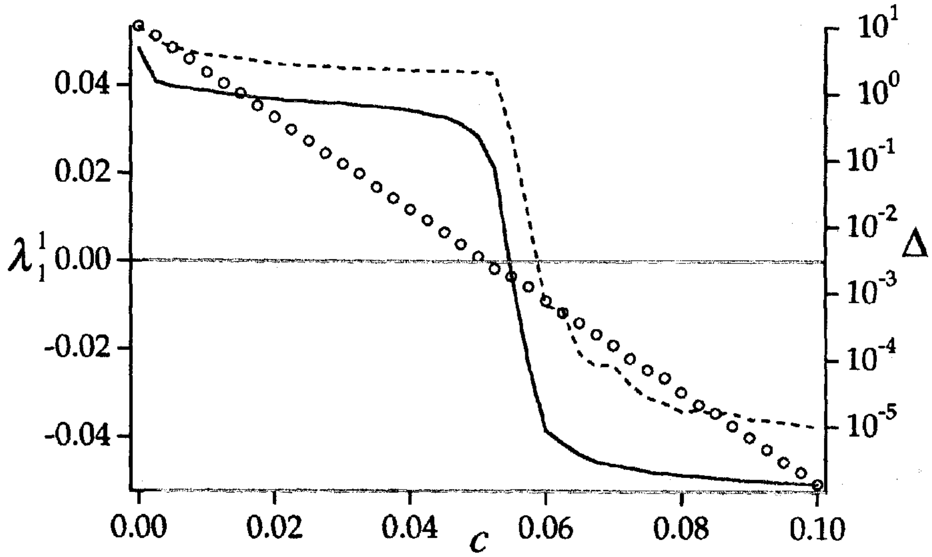


Coupled PL Rossler systems (diffusive coupling constant = c)

Transverse Lyapunov exponents for different UPO's



Rossler system synchronization (2 coupled Rosslers)

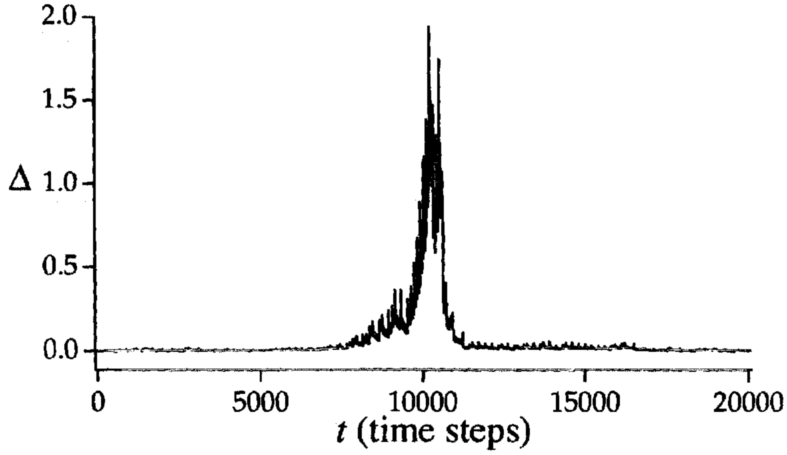


o Maximum transverse Lyapunov exponent

—— RMS difference

— — — Peak difference

Single burst

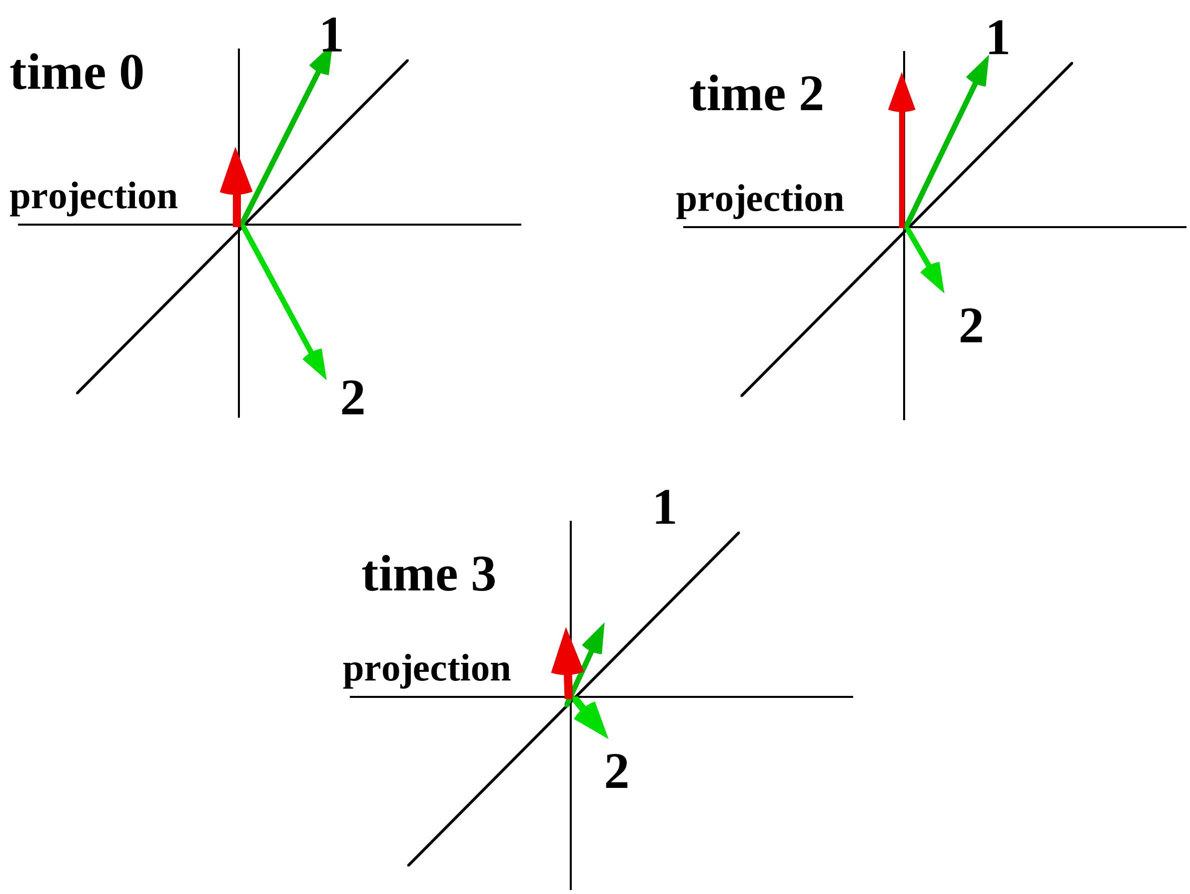


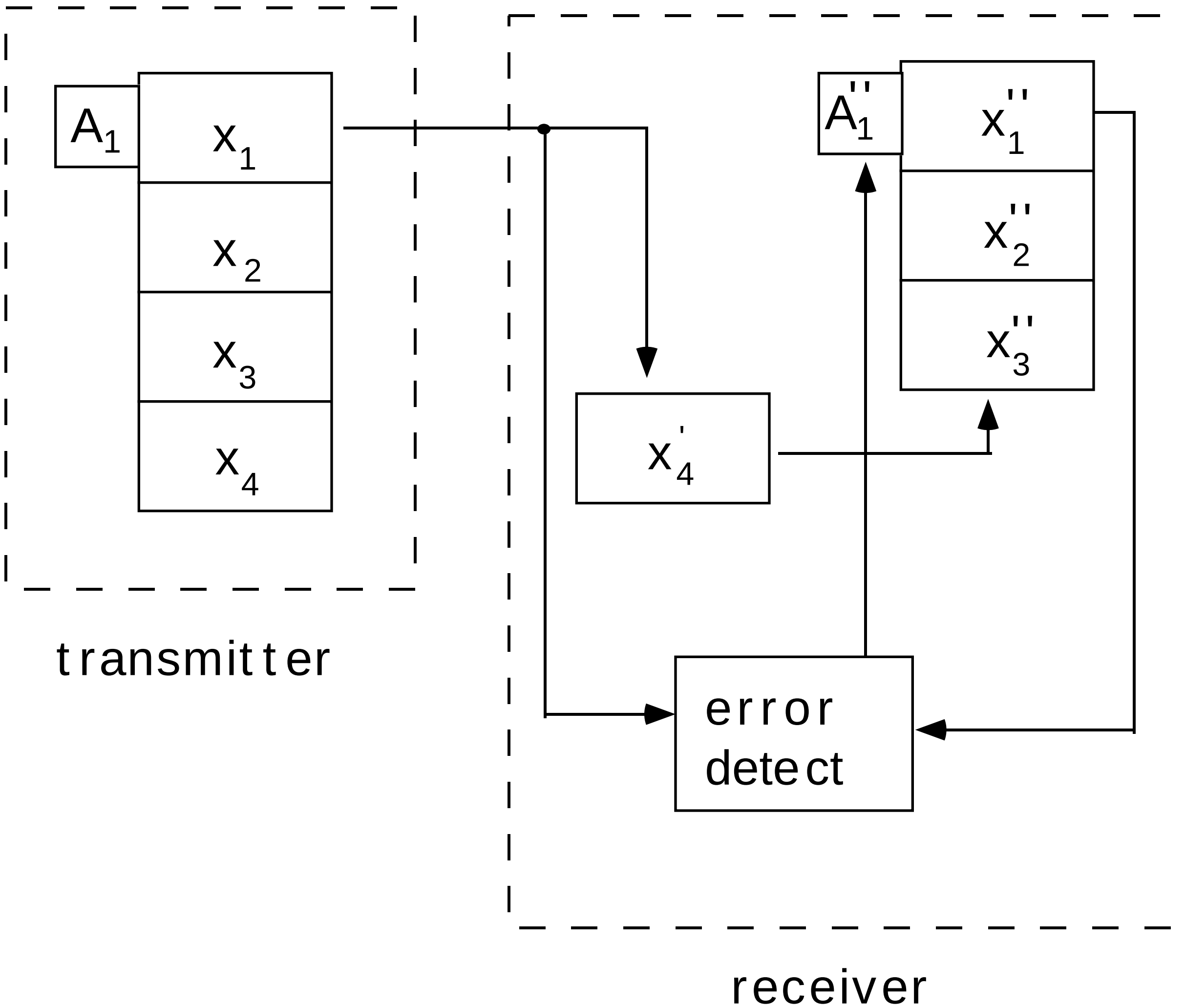
Other complications in synchronization:

Diagonalize Jacobian:

$$\begin{bmatrix} \frac{d\zeta_1}{dt} \\ \frac{d\zeta_2}{dt} \\ \frac{d\zeta_3}{dt} \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 & 0 \\ 0 & \Lambda_2 & 0 \\ 0 & 0 & \Lambda_3 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix}$$

Eigenvectors may not be orthogonal (non-normal)





A_1
x_1
x_2
x_3
x_4
transmitter
A_1''
x_1''
x_2''
x_3''
x_4'
error detect
receiver

Synchronous Substitution

Send a function of the variables
(similar to Kocarev and Parlitz)

$$\frac{d\vec{x}}{dt} = F\left(x_1, x_2, x_3\right)$$

send w = T(x_1, x_2, x_3)

at response, invert T
For example: $\tilde{x}_2 = T^{-1}\left(w, x'_1, x'_2, x'_3\right)$

$$\frac{d\vec{x}'}{dt} = F\left(x'_1, \tilde{x}_2, x'_3\right)$$

Examples of Synchronous Substitution

PL Rossler circuit:

Transmit $w = y - x$

Then $\tilde{y} = w + x$

Response Jacobian:

$$\begin{bmatrix} -0.05 & -0.5 & -1.0 \\ 1.0+1.0 & 0-0.02 & 0 \\ \frac{\partial g(x)}{\partial x} & 0 & -1 \end{bmatrix}$$

$\lambda_{max} = -196\ s^{-1}$

Synchronous substitution examples

Send $w = \frac{-y}{(x+4.2)}$

Then $\tilde{y} = -w(x' + 4.2)$

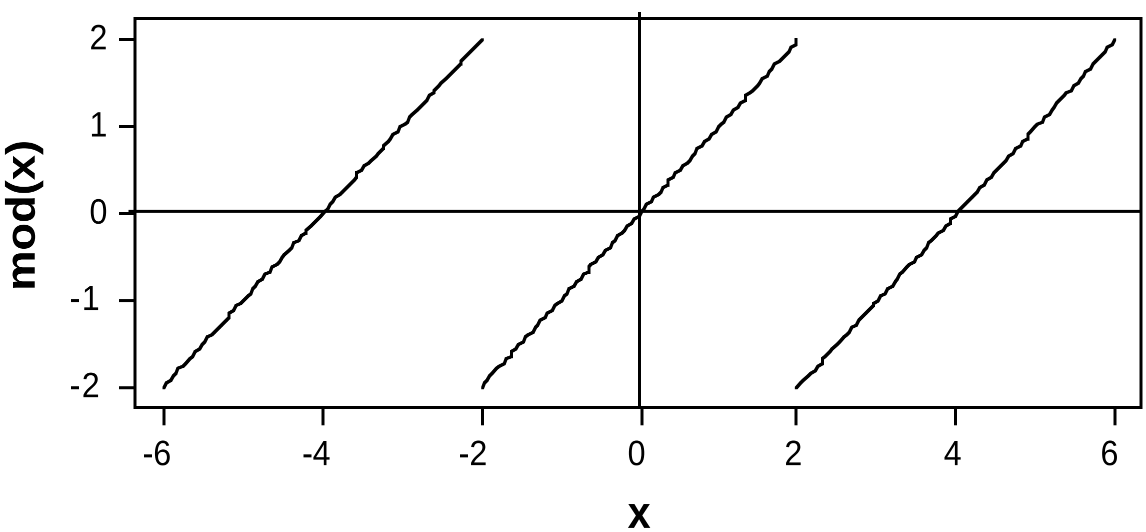
Response Jacobian:

$$\begin{bmatrix} -0.05 & -0.5 & -1.0 \\ 1.0 - w & 0 - 0.02 & 0 \\ \frac{\partial g(x)}{\partial x} & 0 & -1 \end{bmatrix}$$

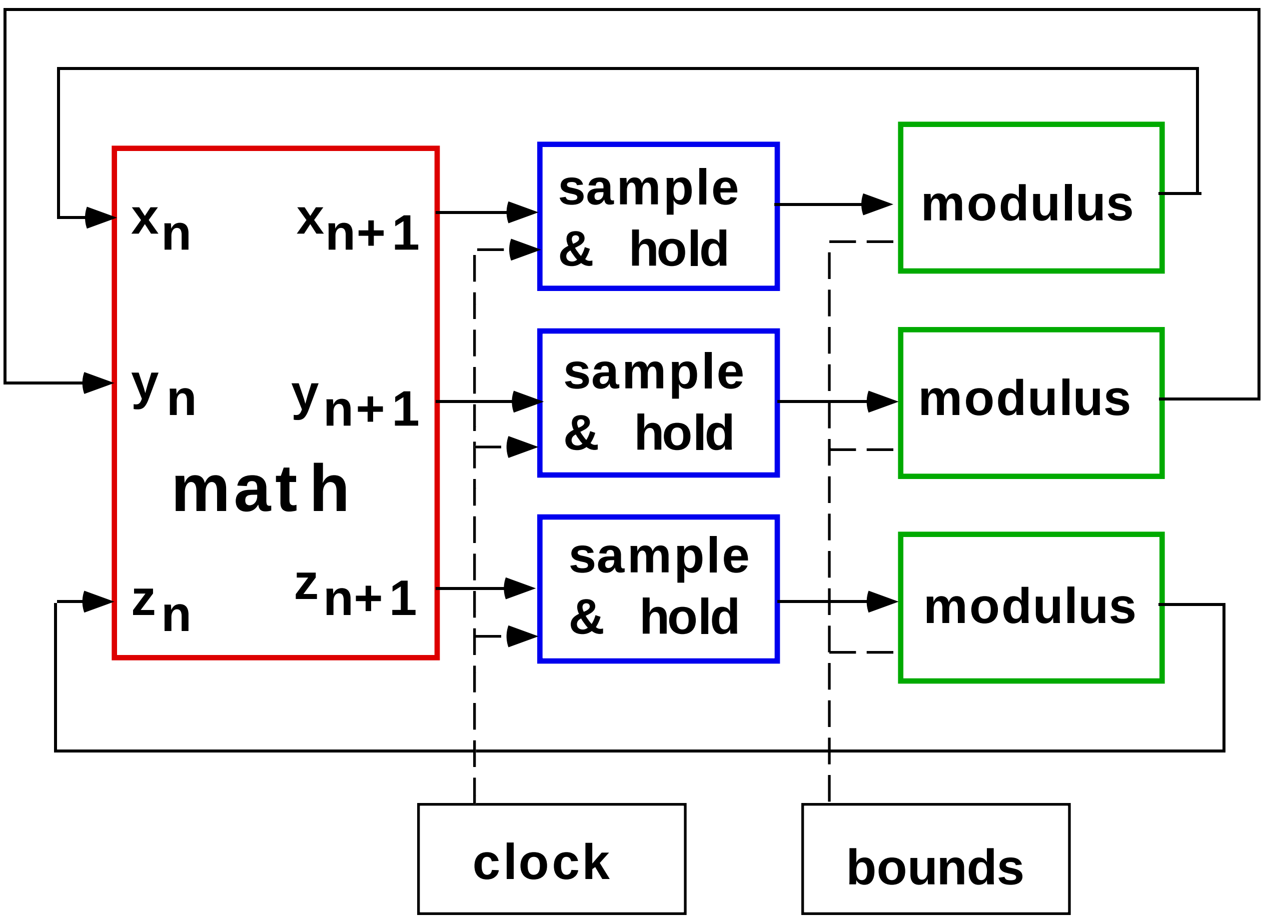
λ_{max}**=-651 s**$^{-1}$

Synchronizing Volume-Preserving Maps

$$\left\{\begin{array}{l} x_{n+1} = -\frac{4}{3}x_n + z_n \\ y_{n+1} = \frac{1}{3}y_n + z_n \\ z_{n+1} = x_n + y_n \end{array}\right\} \bmod \pm 2$$

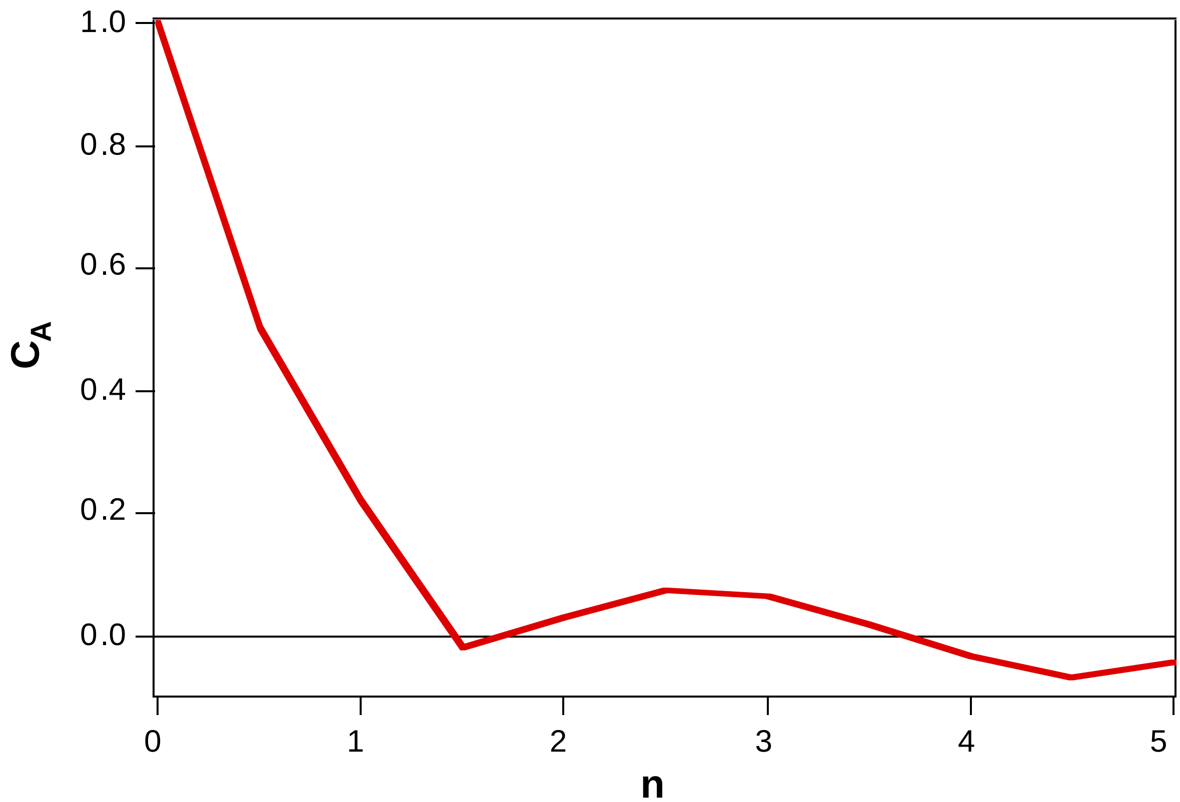


Circuit block diagram

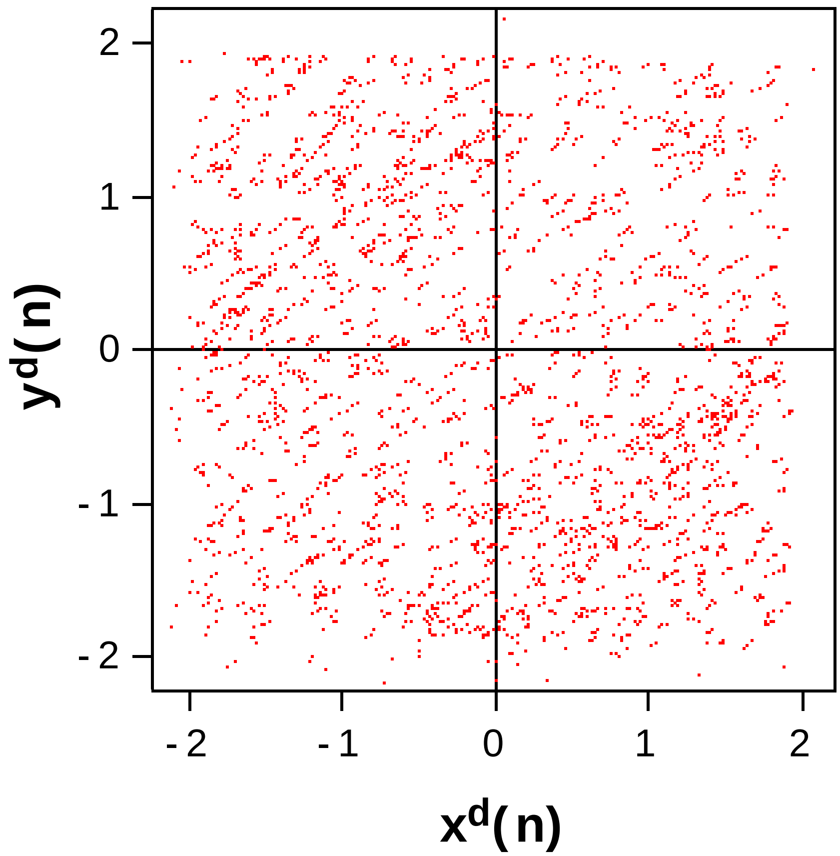


Lyapunov exponents:
0.6829 + 0.3023 + (-0.9852) = 0

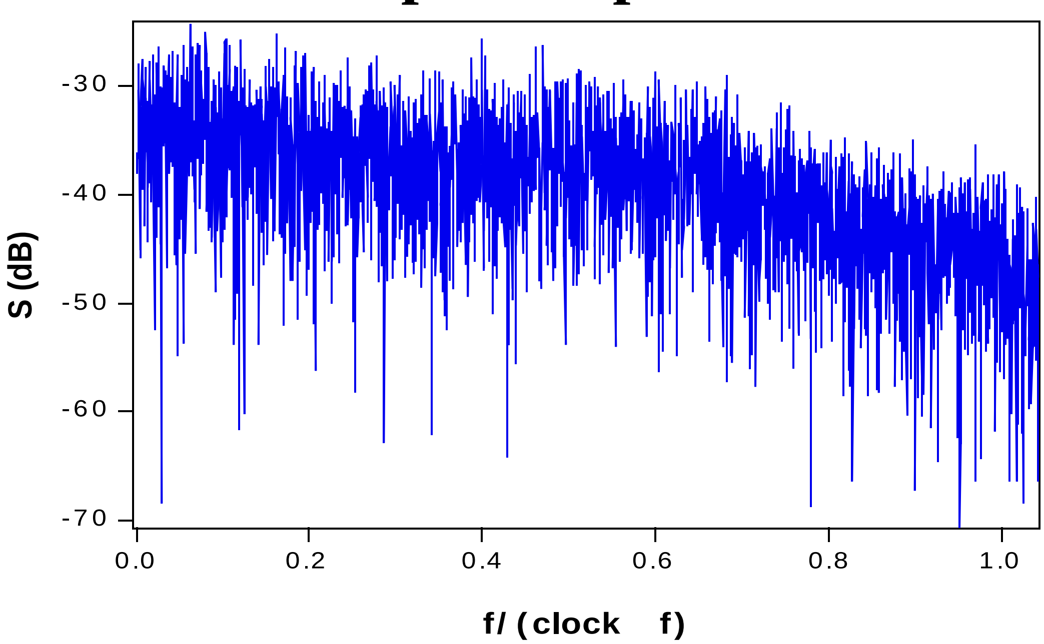
Autocorrelation (from circuit)



Circuit attractor



Circuit power spectrum



How to couple circuits?

Drive Jacobian:

$$\begin{bmatrix} -\frac{4}{3} & 0 & 1 \\ 0 & \frac{1}{3} & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

transmit

$$\boldsymbol{w}_{\mathbf{n}} = \boldsymbol{z}_{\mathbf{n}} + \Gamma \boldsymbol{x}_{\mathbf{n}}$$

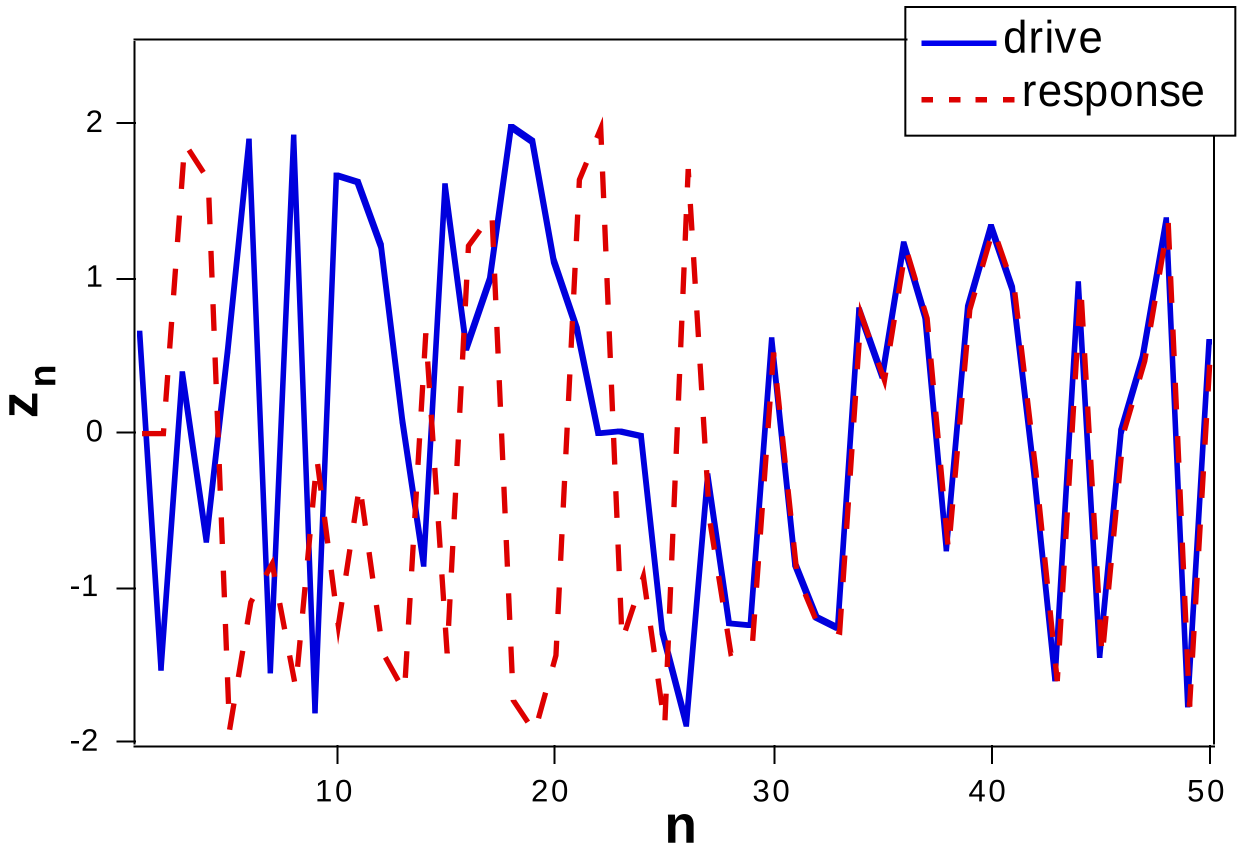
drive x-y subsystem with:

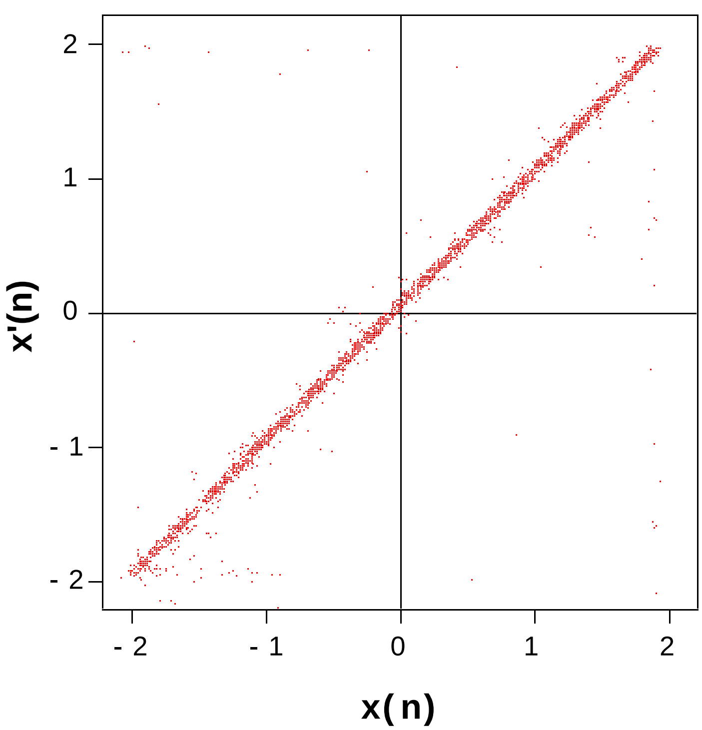
$$\hat{z}_n = w_n - \Gamma x'_n$$

$$\begin{bmatrix} -\frac{4}{3}-\Gamma & 0 \\ -\Gamma & -\frac{1}{3} \end{bmatrix}$$

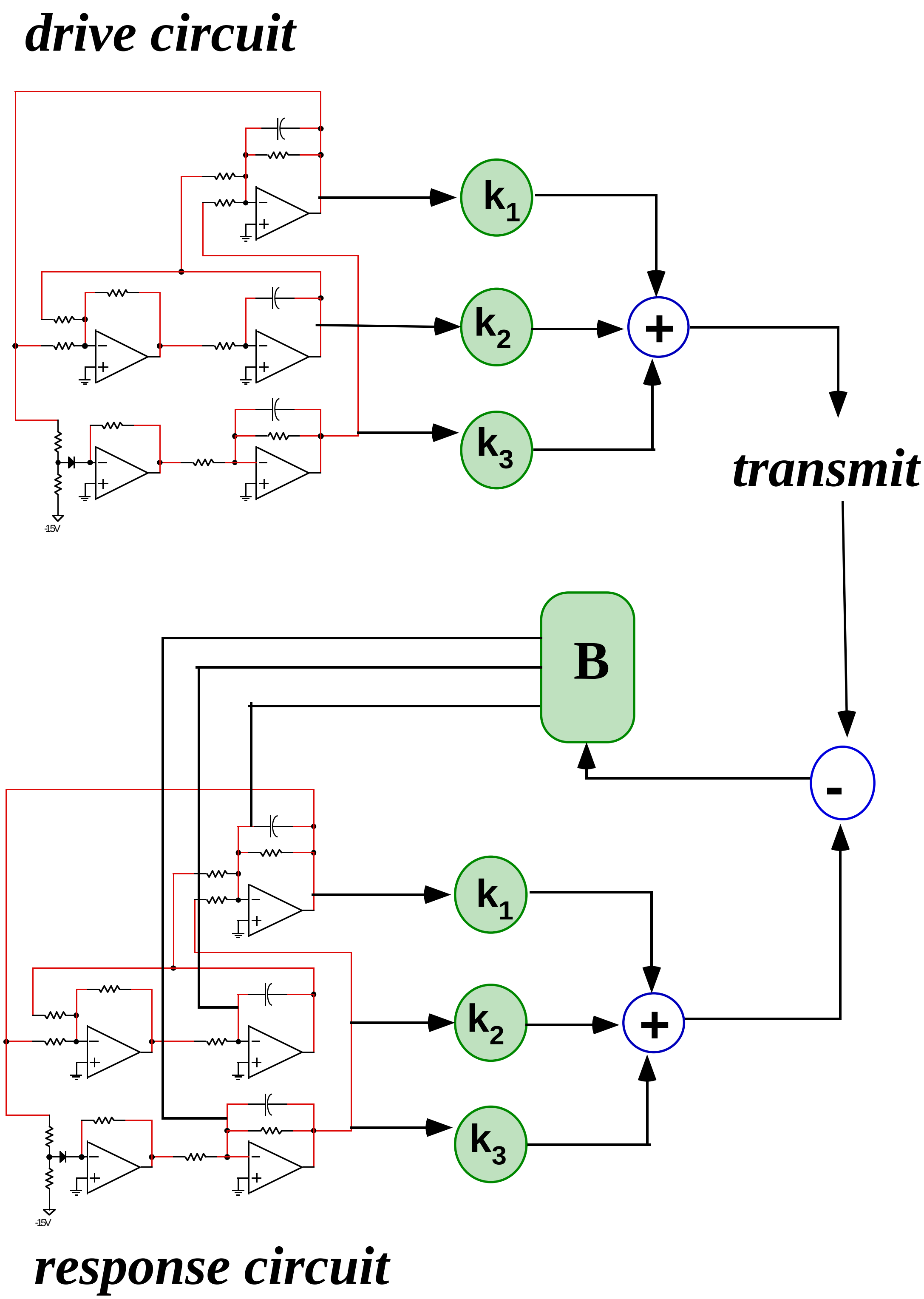
Use Γ = -4/3

Map synchronization





General coupling scheme
(Peng et al. PRL 1996)



General coupling:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{F}[\mathbf{x}(t)]$$

$$\frac{d\mathbf{y}(t)}{dt} = \mathbf{F}[\mathbf{y}(t)] - \mathbf{B}\mathbf{K}^{\mathbf{T}}[\mathbf{y}(t) - \mathbf{x}(t)]$$

Jacobian for PL Rossler

$$\begin{bmatrix} -0.05 - k_1 b_1 & -0.5 - k_2 b_1 & -1.0 - k_2 b_1 \\ 1.0 - k_1 b_2 & 0.131 - k_2 b_2 & 0 - k_3 b_2 \\ \frac{\partial g(x)}{\partial x} - k_1 b_3 & 0 - k_2 b_3 & -1 - k_3 b_3 \end{bmatrix}$$

General coupling

For n dimensions, have 2n parameters

$k_1, \ldots k_n; \quad b_1, \ldots b_n$

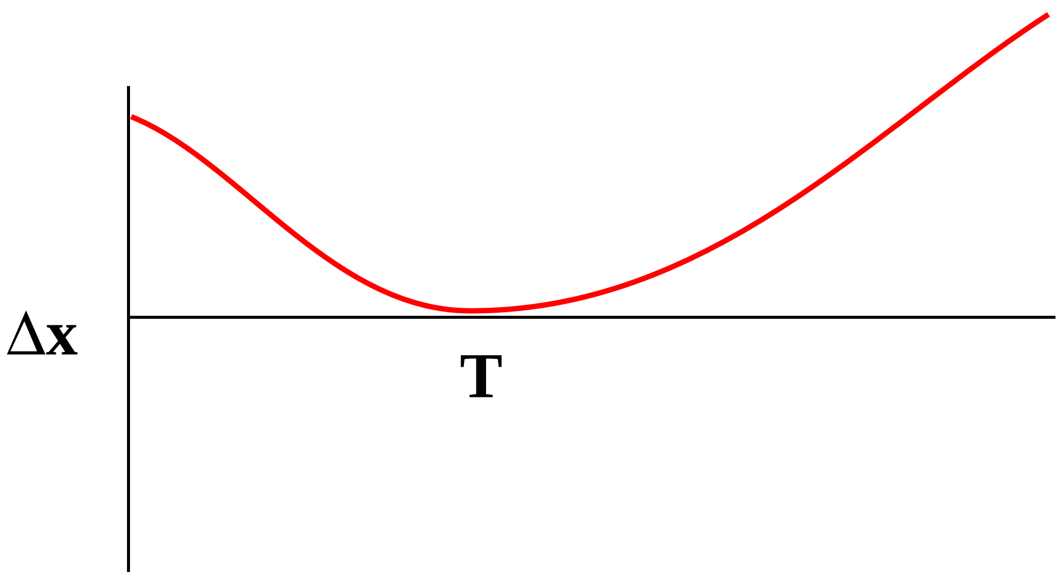
Use minimization algorithm to minimize largest response Lyapunov exponent by varying k's and b's.

Many minima.

Pitfalls of minimization

Calculate λ for interval of length T:
Algorithm will minimize expansion
over this interval

Possible result



Actual system is unstable, but calculated λ is negative

Hyperchaotic Rossler circuit equations

(design by Gregg Johnson)

$$dx_1/dt = -.05x_1 - .5x_2 - .62x_3$$

$$dx_2/dt = x_1 + \rho x_2 + .40x_4$$

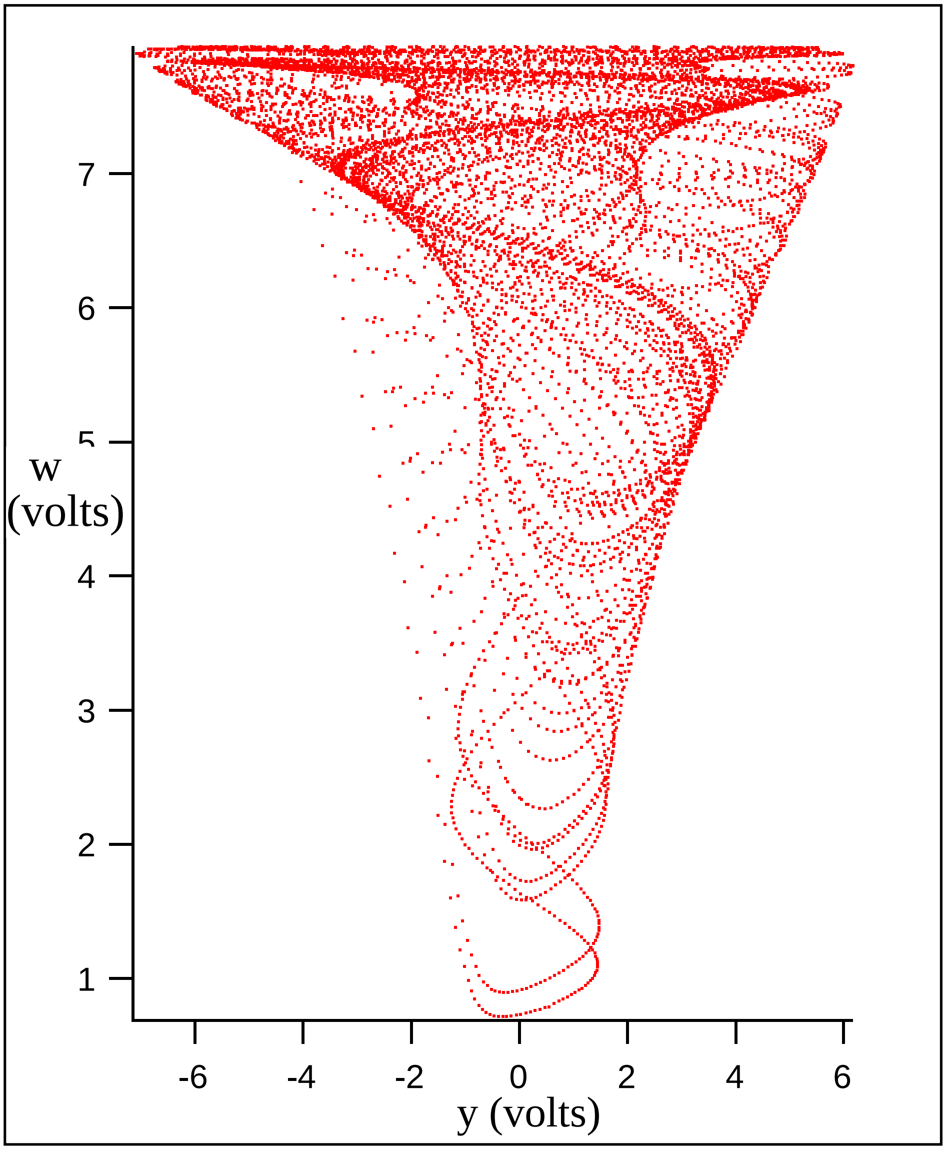
$$dx_3/dt = -2x_3 + g(x_1)$$

$$dx_4/dt = -1.5x_3 + .18x_4 + h(x_4)$$

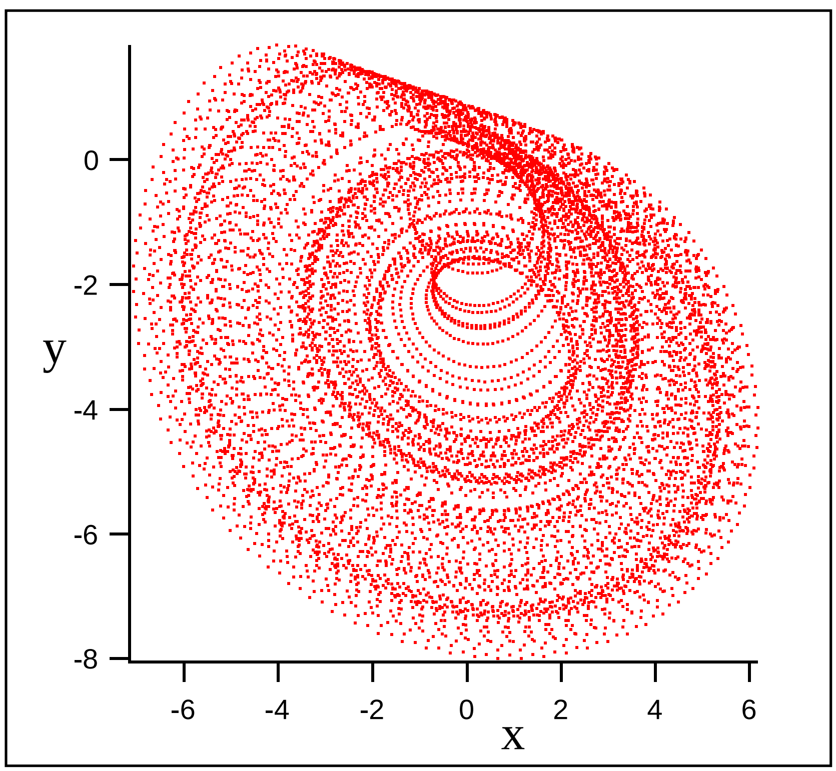
Piecewise linear functions:

$$g(x_1) = 10(x_1 - .68)\,\Theta(x_1 - .68)$$

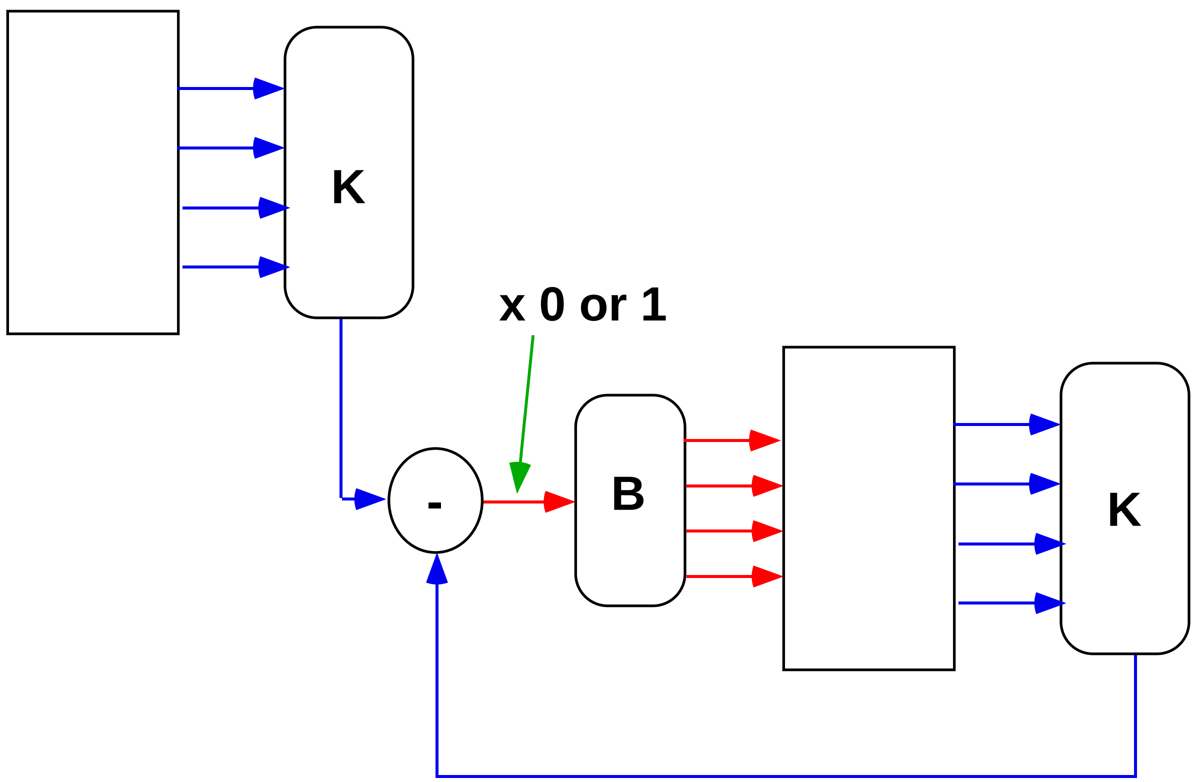
$$h(x_4) = -.41(x_4 - 3.8)\,\Theta(x_4 - 3.8)$$



Circuit Attractors (experiment)



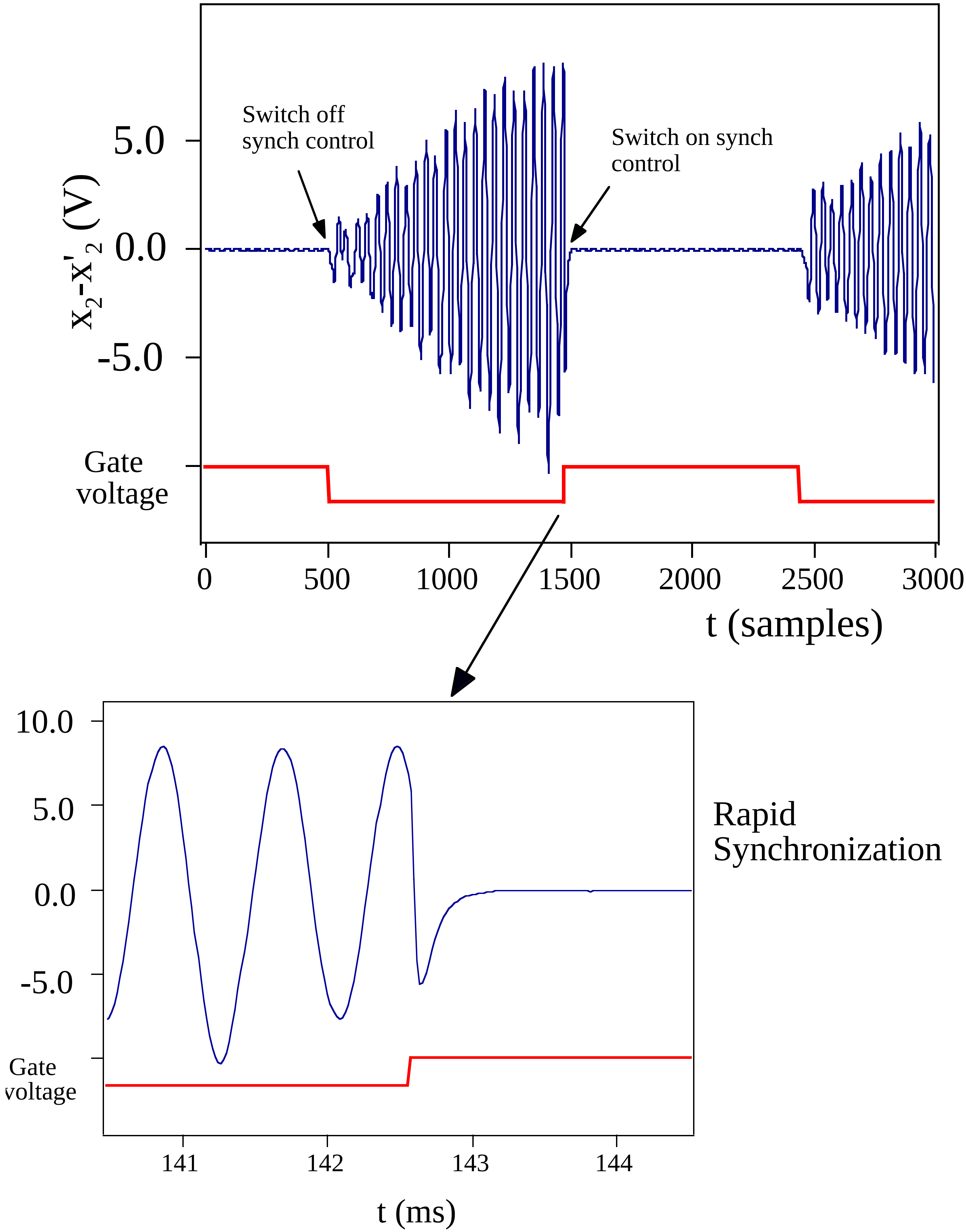
Coupling coefficients for 4-D Rossler



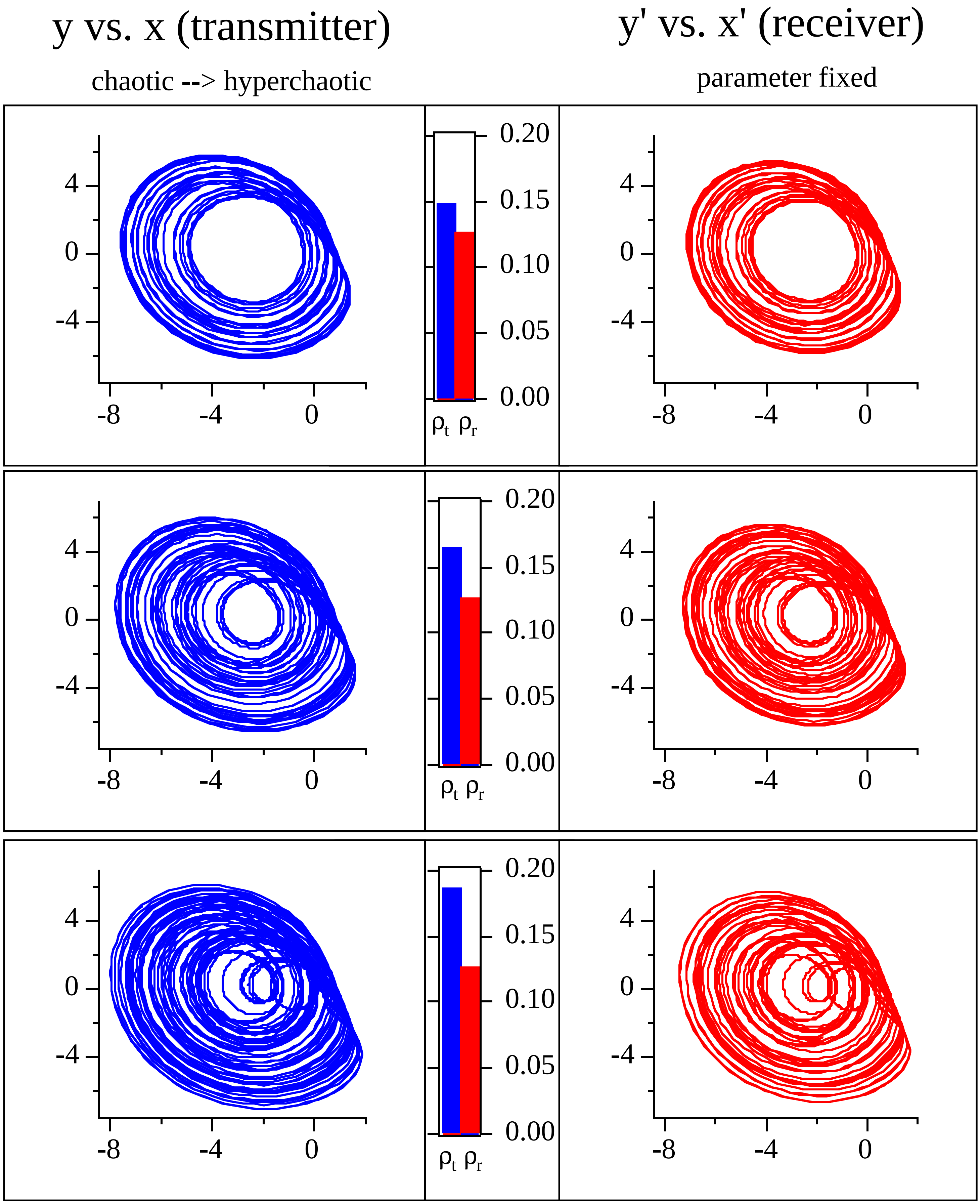
$$\mathbf{K} = (-1.97, 2.28, 0, 1.43)$$

$$\mathbf{B} = (0.365, 2.04, -1.96, 0)$$

$$\lambda = -0.34$$



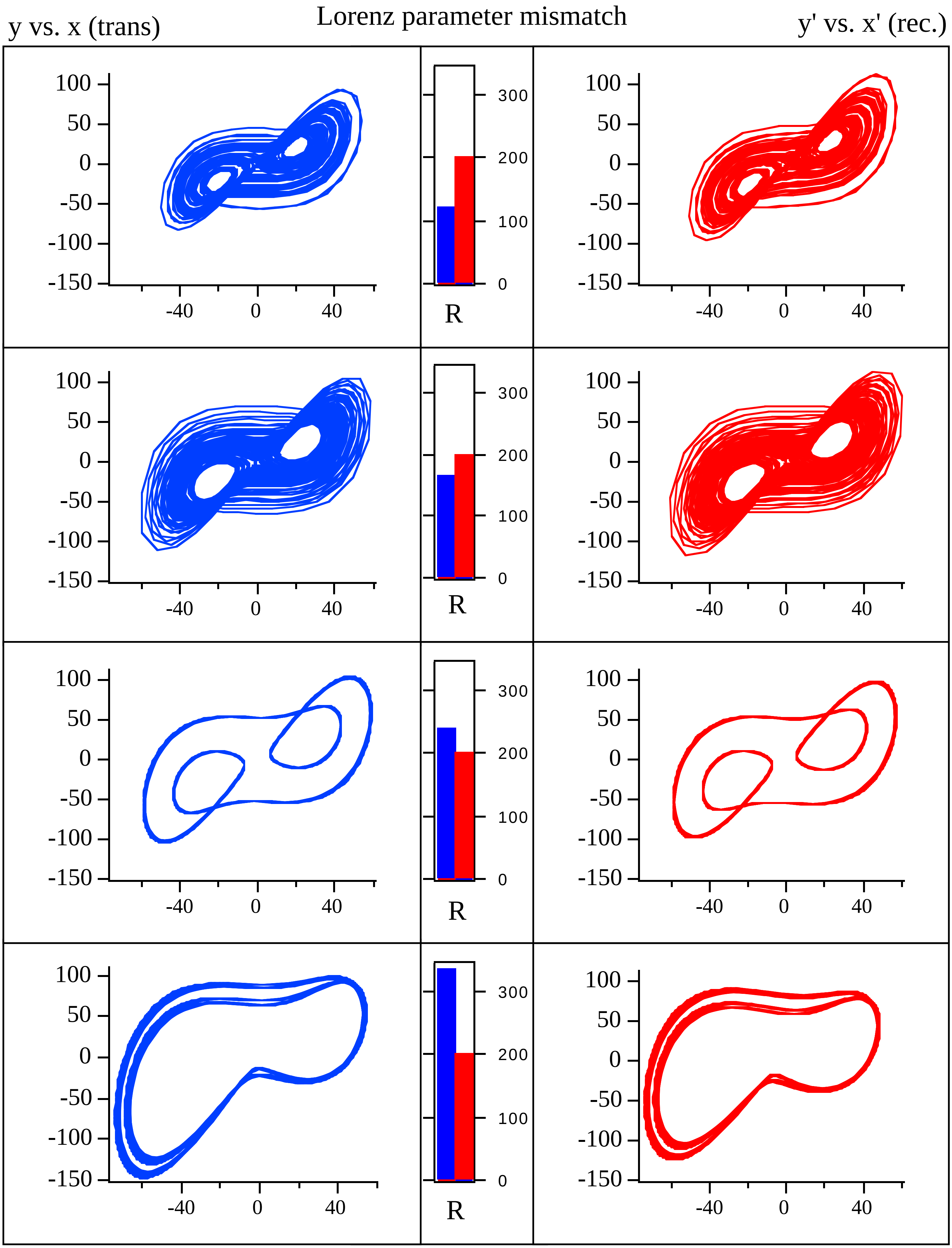
Switch off
synch control
Switch on synch
control
5.0
0.0
-5.0
$x_2 - x'_2$ (V)
Gate
voltage
0
500
1000
1500
2000
2500
3000
t (samples)
10.0
5.0
0.0
-5.0
Gate
voltage
141
142
143
144
t (ms)
Rapid
Synchronization



Experiment: 4D Rossler circuit, ρ_r fixed

Lorenz equations

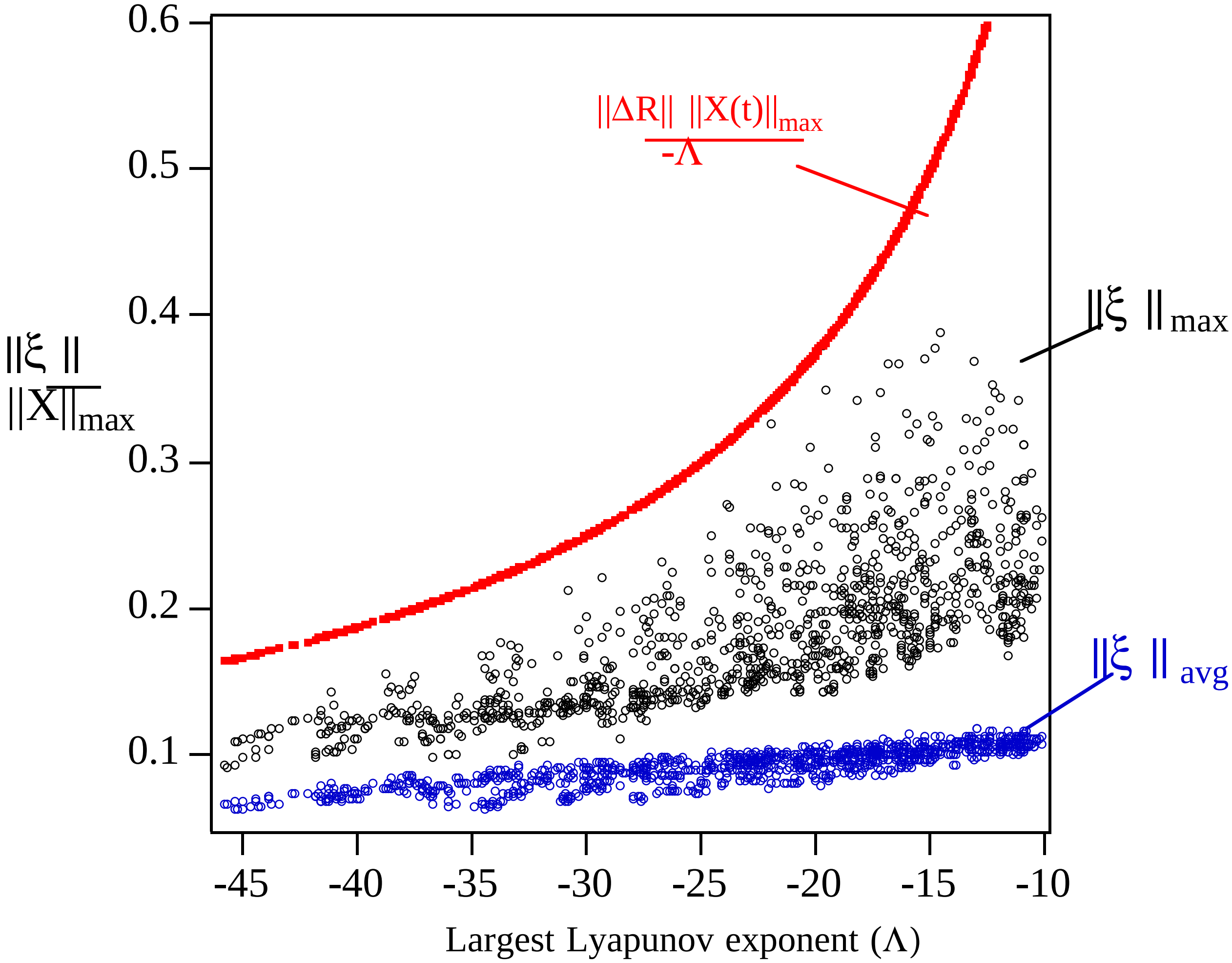
$$dx_1 / dt = 16(x_2 - x_1)$$
$$dx_2 / dt = x_1(R - x_3) - x_2$$
$$dx_3 / dt = x_1 x_2 - 4x_3$$



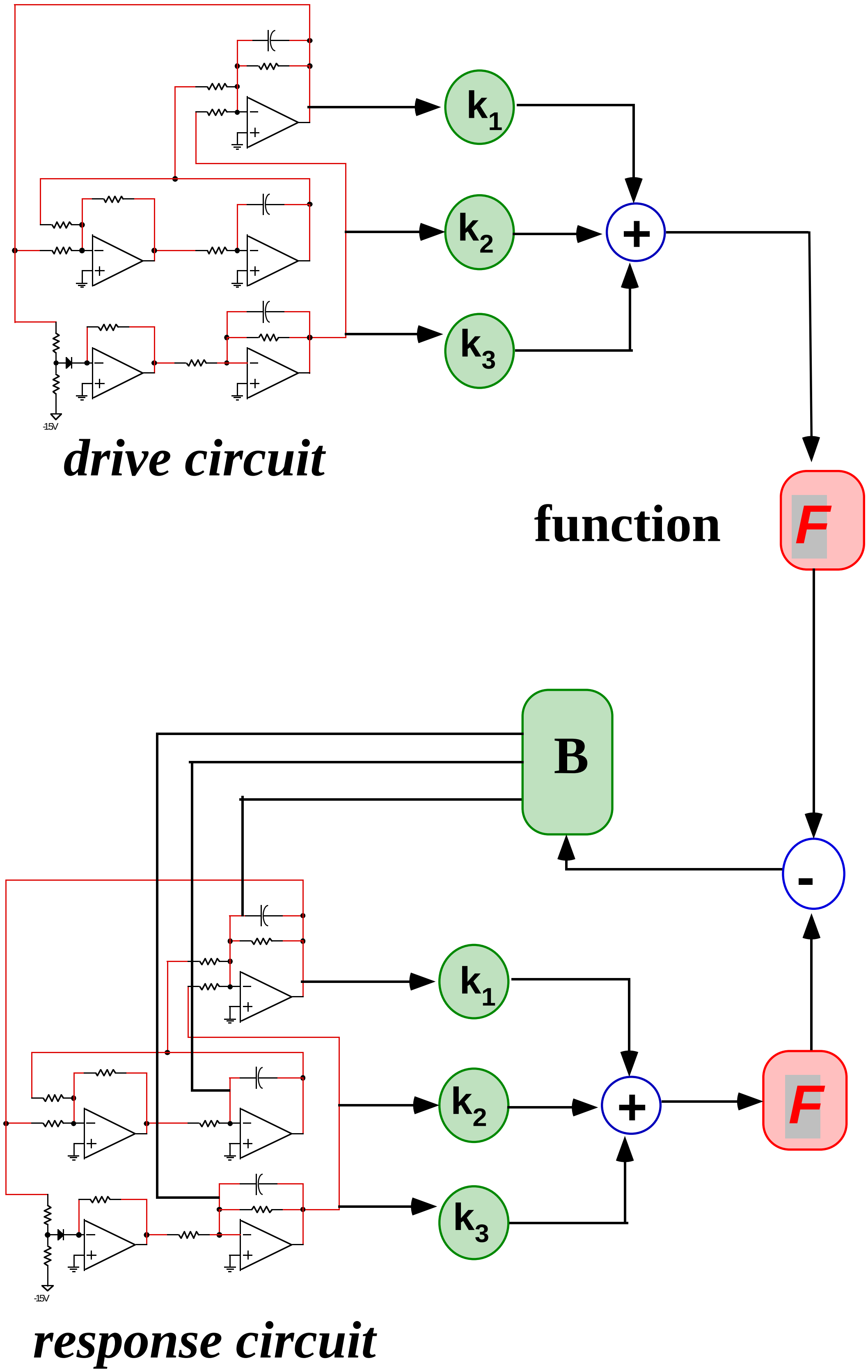
Lorenz parameter mismatch
y vs. x (trans)
y' vs. x' (rec.)
R

Lorenz system

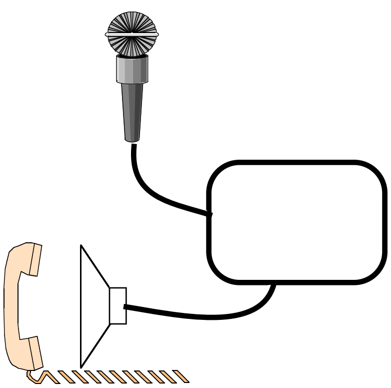
ΔR = 17.5 %
R_T = 165
R_R = 200

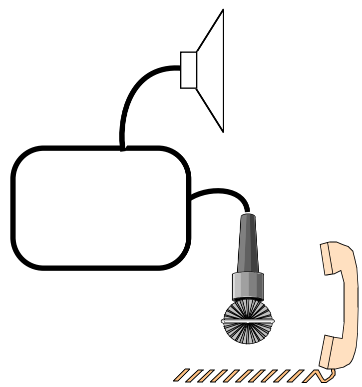


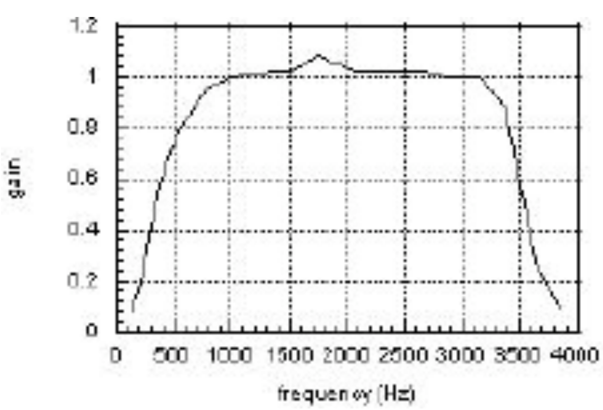
Add function to circuits



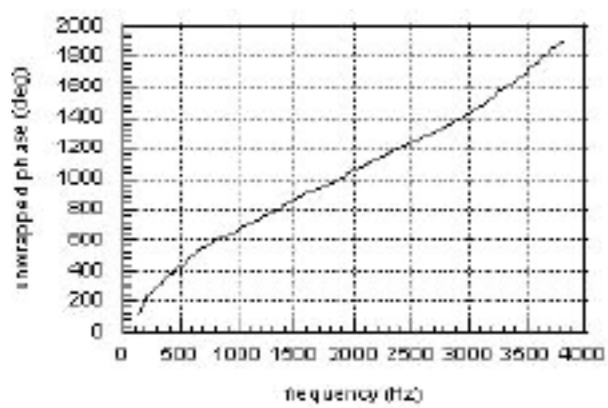
Telephone Scrambler







amplitude

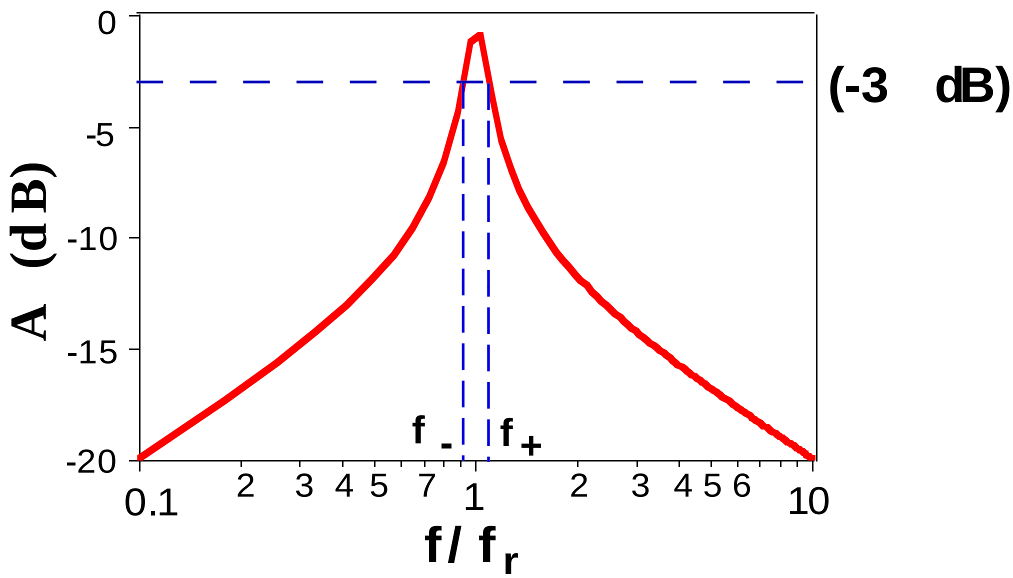


phase

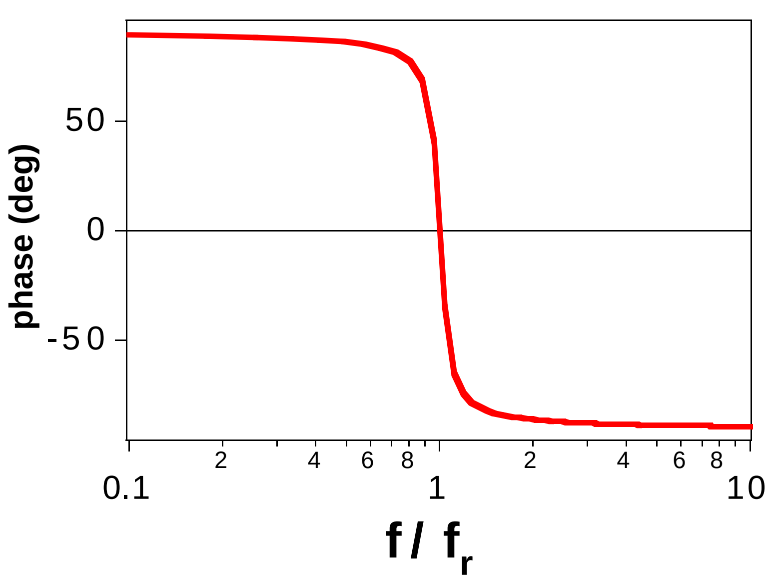
Use a bandpass filter for *F*

A bandpass filter is a second order (or higher) dynamical system.

Second order bandpass filter characteristics:



$$Q = f_r/(f_+ - f_-)$$

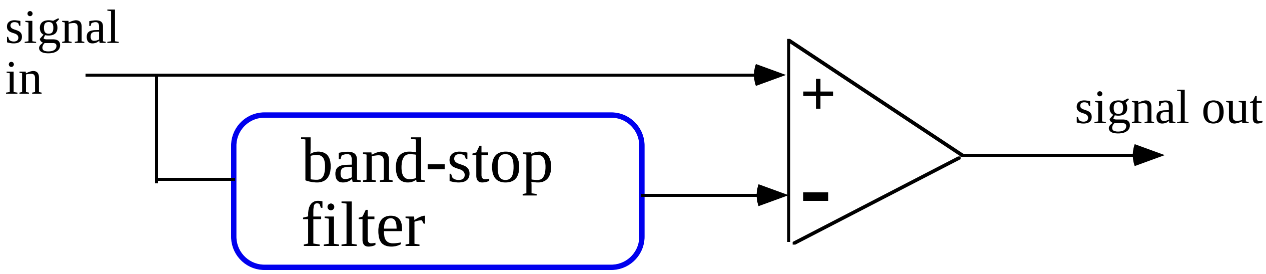


Form of the filter F is important

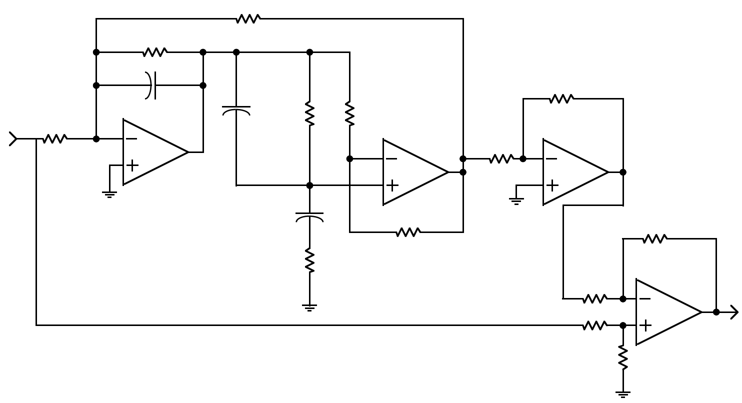
If output of F does not explicitly contain x, y, or z, then Jacobian looks like:

$$\begin{bmatrix} J_{11} & J_{12} & J_{13} & \dfrac{\partial F(x,y,z,K,B)}{\partial x} \\ J_{21} & \cdots & & \\ \vdots & \ddots & & \\ & & & \end{bmatrix}$$

Filtering process for Rossler signal



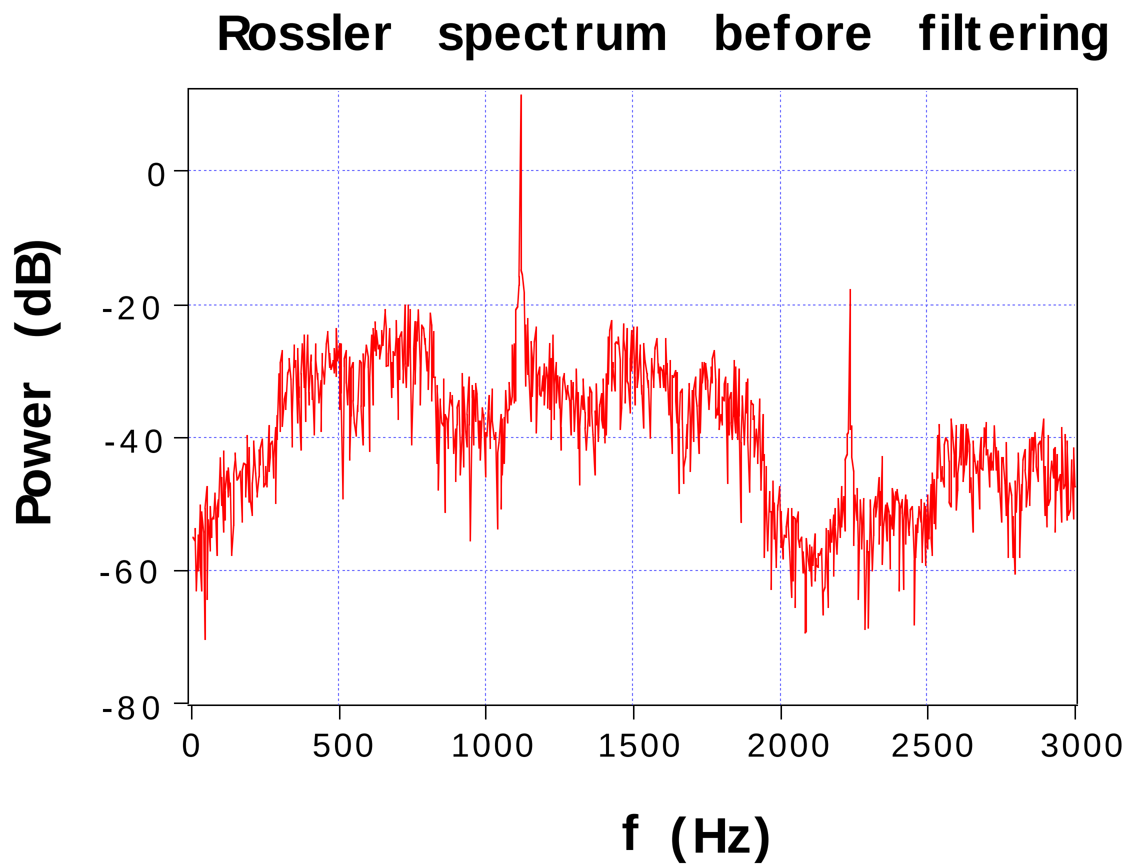
Wien-Robinson bandstop filter

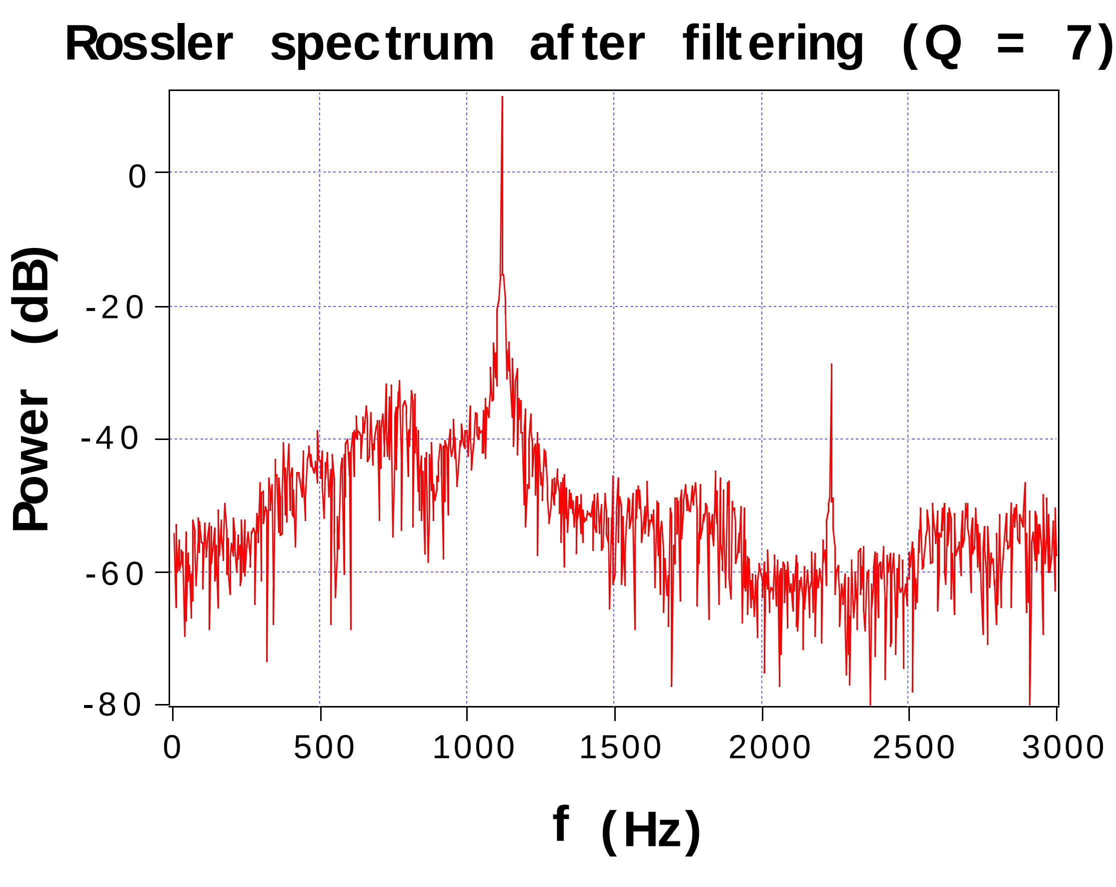


g_2 = output, u = input

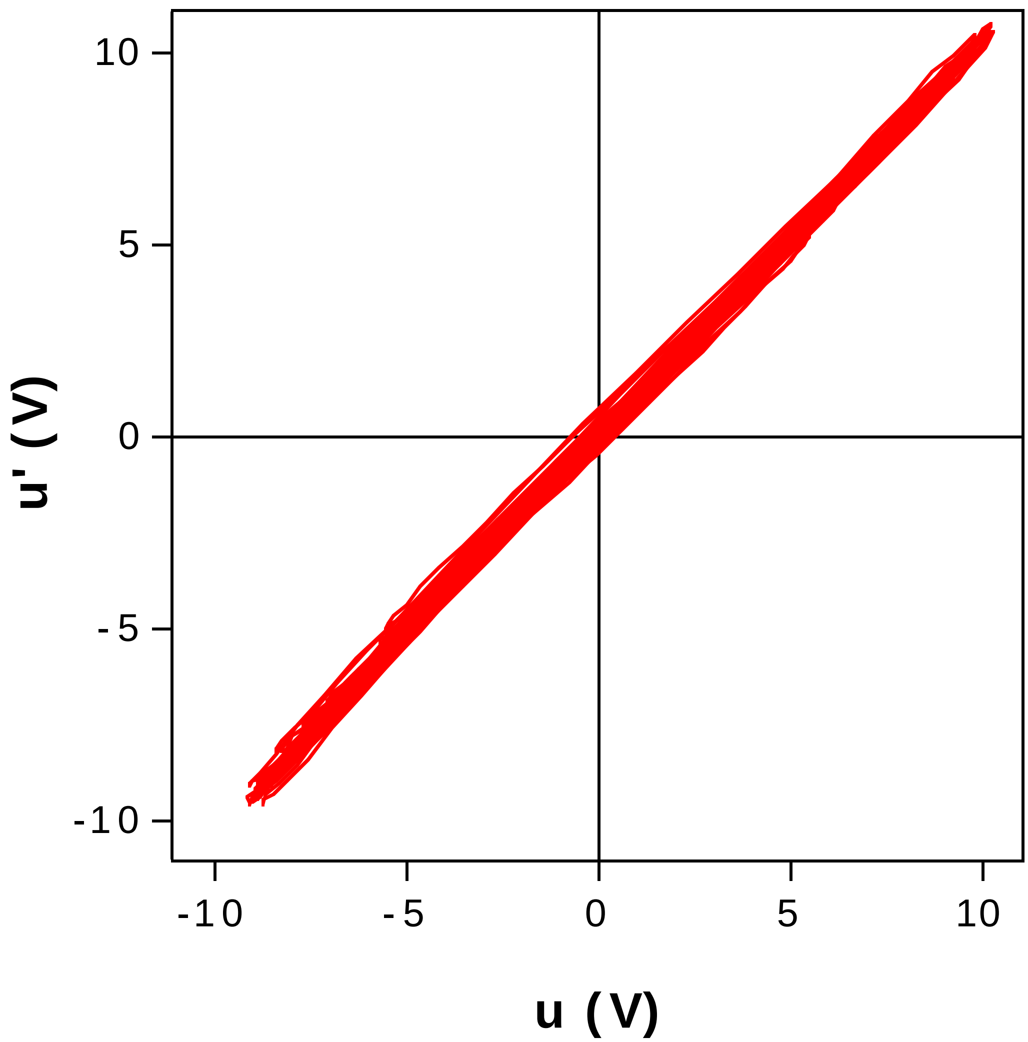
$$\frac{dg_1}{dt} = \frac{1}{RC}\left(A_0 u - g_2 - \frac{1}{Q} g_1\right) + A_0 RC \frac{d^2 u}{dt^2}$$

$$\frac{dg_2}{dt} = \frac{1}{RC} g_1$$

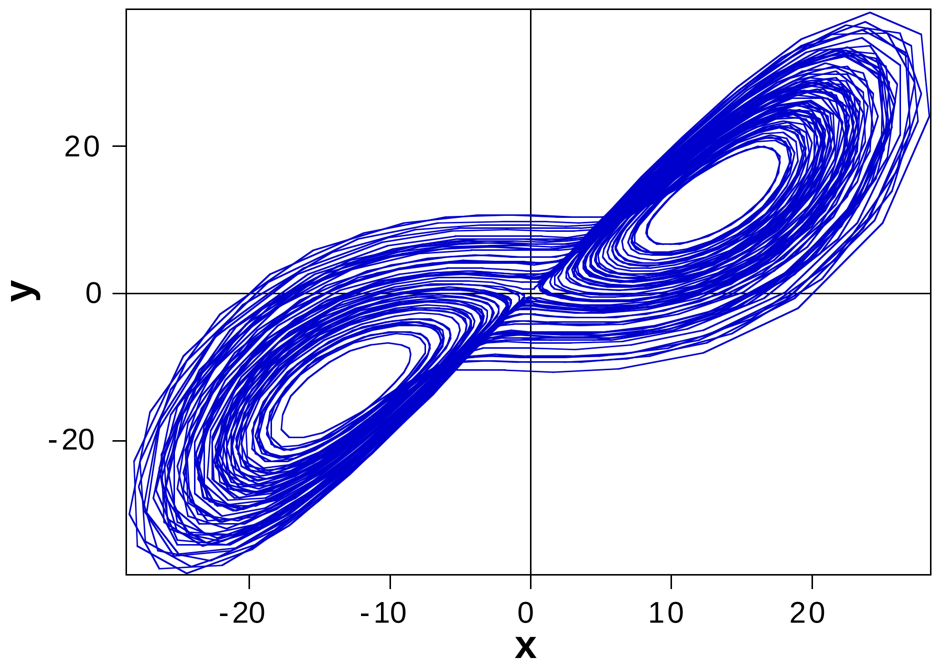

Rossler spectrum before filtering
Power (dB)
0
-20
-40
-60
-80
0
500
1000
1500
2000
2500
3000
f (Hz)


Rossler spectrum after filtering (Q = 7)
Power (dB)
0
-20
-40
-60
-80
0
500
1000
1500
2000
2500
3000
f (Hz)

Rossler synchronization: u' vs u



Lorenz system



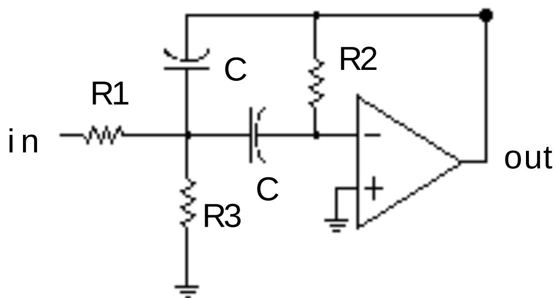
$$\frac{dx}{dt} = 16(y - x)$$

$$\frac{dy}{dt} = -xz + 45.92x - y$$

$$\frac{dz}{dt} = xy - 4z$$

Filtering process for Lorenz system

bandpass filter

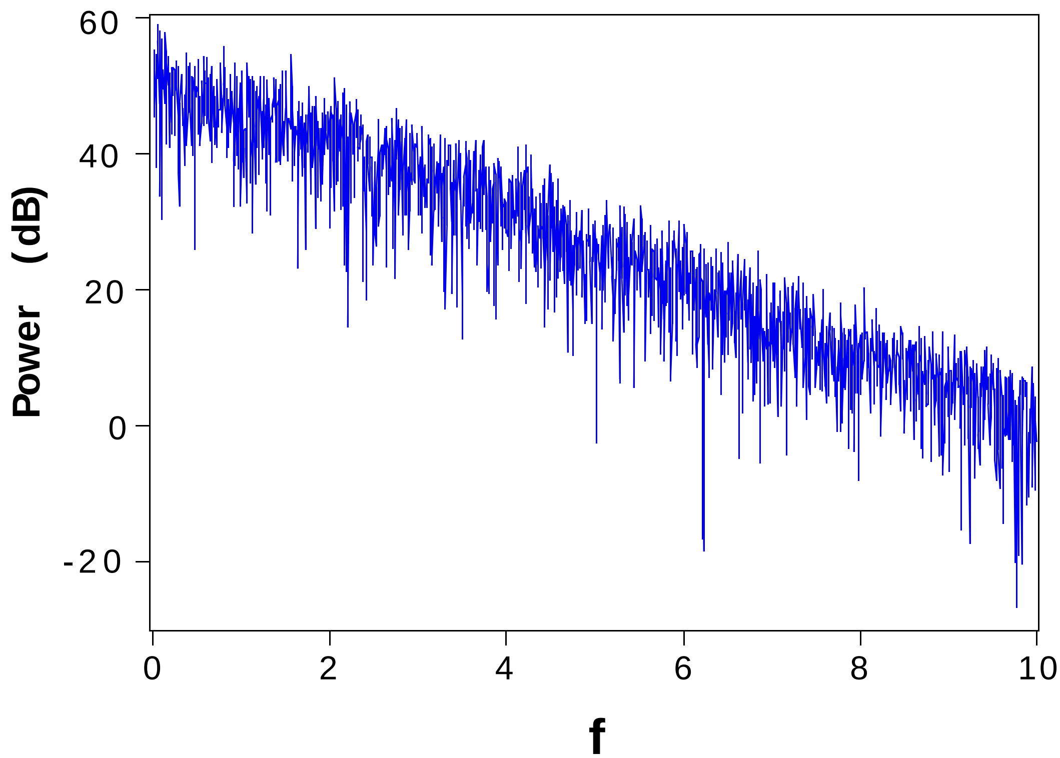


u = input, g_2 = output

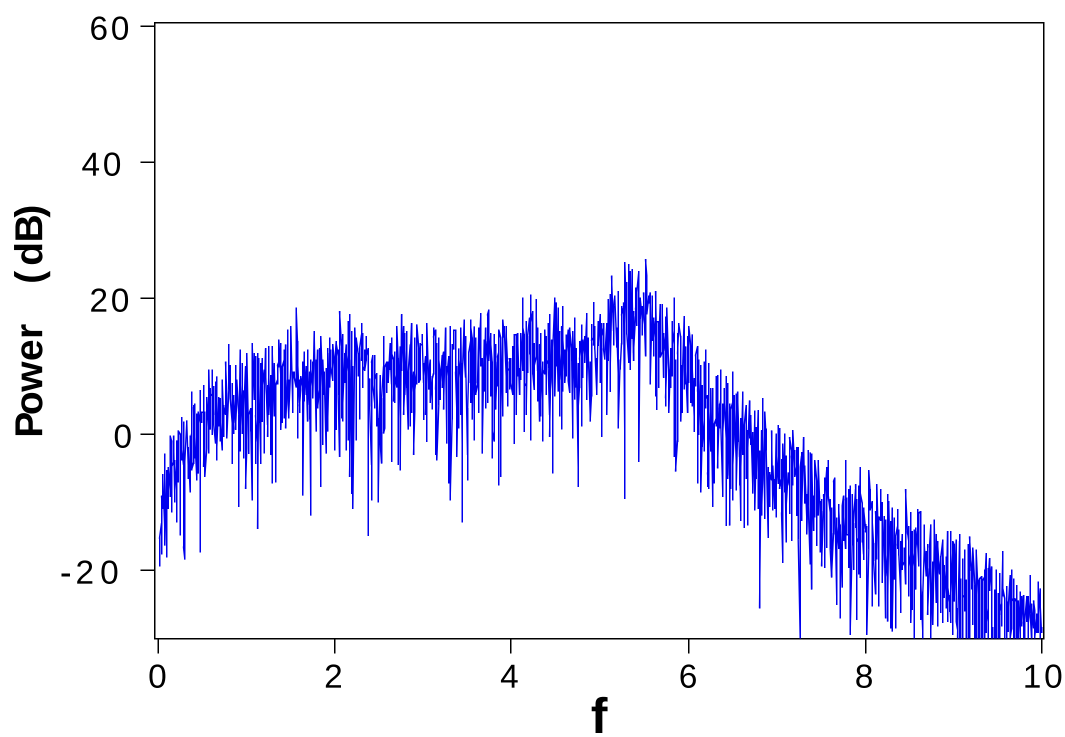
$$\frac{dg_1}{dt}=\frac{-2}{R_1C}g_1-\left(\frac{1}{2R_2C}\right)\left(\frac{1}{R_3C}-\frac{1}{R_1C}\right)g_2-\left(\frac{1}{R_1C}\right)\frac{du}{dt}$$

$$\frac{dg_2}{dt}=g_1$$

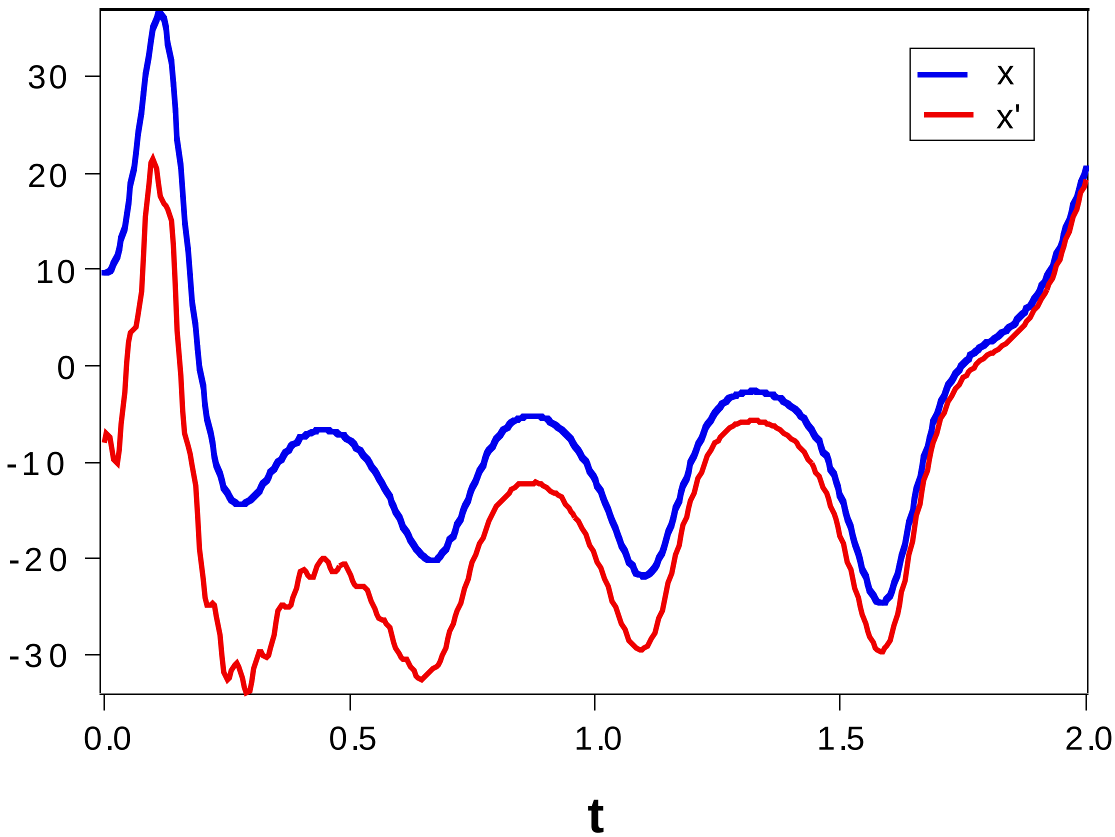
Lorenz signal before filtering



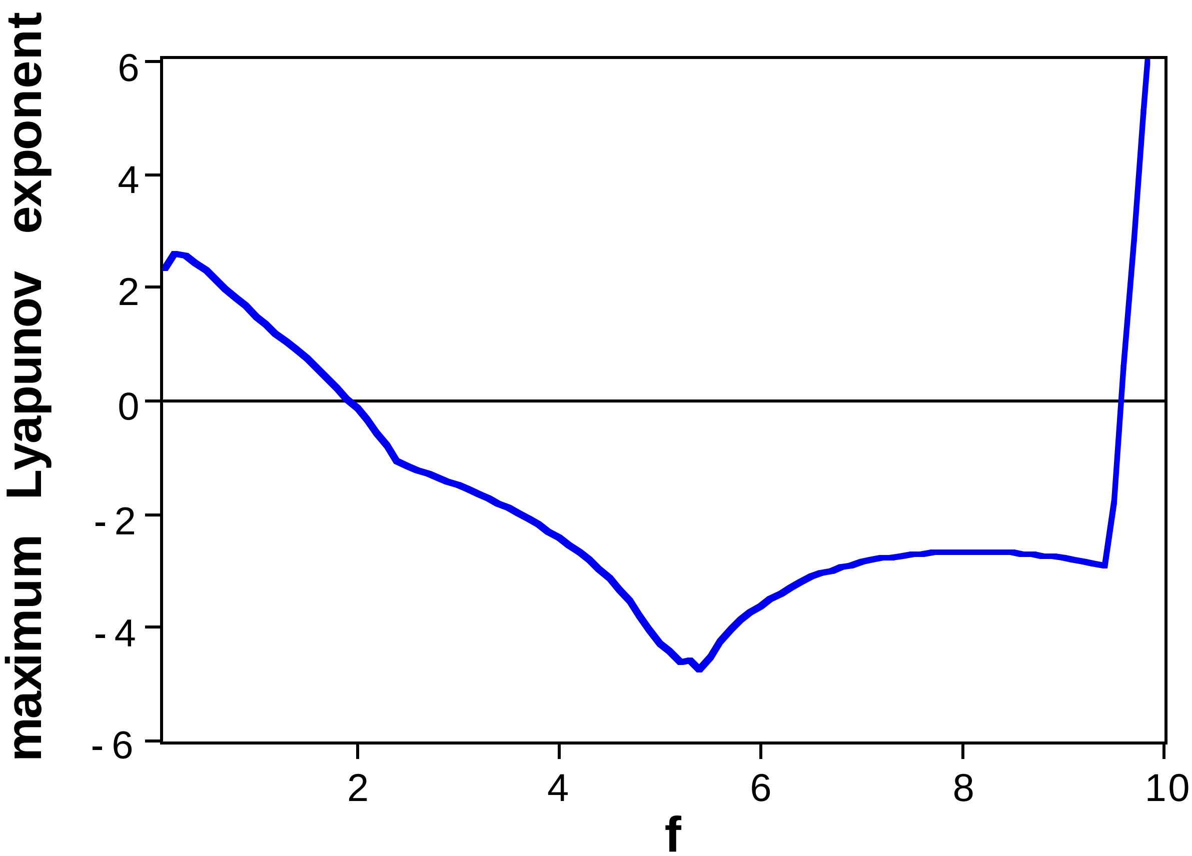
Lorenz signal after filtering



Lorenz synchronization

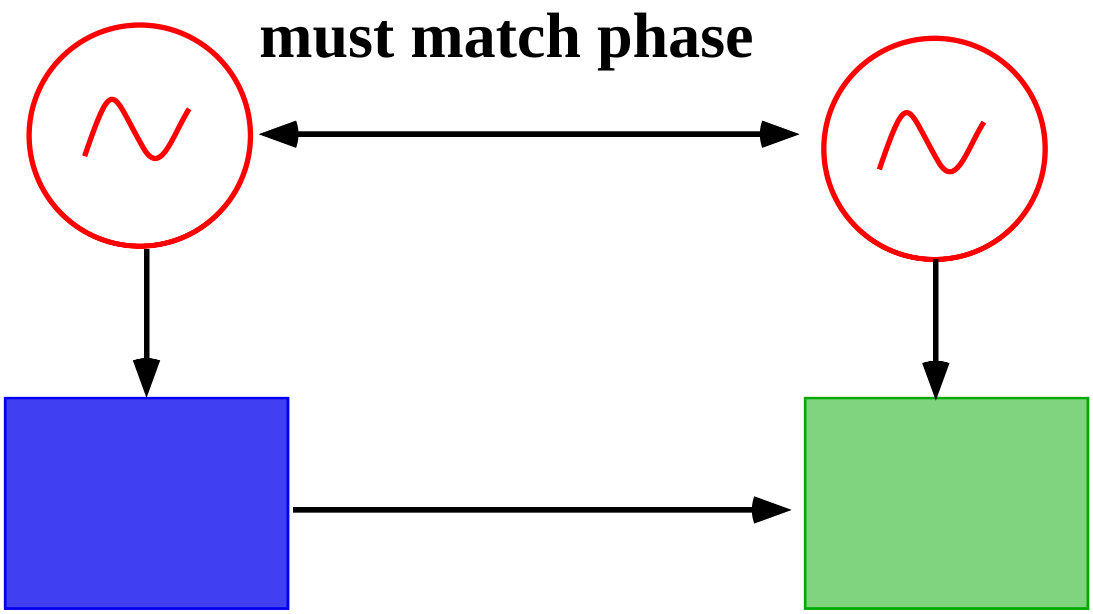


Lorenz stability vs filter center frequency



Other Applications for Filters:

Synchronizing Nonautonomous Circuits



Consider 2 periodically driven systems such as the Duffing system: (Heagy)

$$\frac{dx_1}{dt} = v_1$$
$$\frac{dv_1}{dt} = -\gamma v_1 + g(x_1) + A\cos(\phi_1)$$
$$\frac{d\phi_1}{dt} = \omega$$

$$\frac{dx_2}{dt} = v_2$$
$$\frac{dv_2}{dt} = -\gamma v_2 + g(x_2) + A\cos(\phi_2)$$
$$\frac{d\phi_2}{dt} = \omega$$

Suppose we drive system 2 with the x component of system 1:

$$\dot{v}_2 = -\gamma v_2 + g(x_1) + A\cos\phi_2$$

We will want $\phi_2 - \phi_1$ to go to 0, but we only know x_2 and x_1.

Heagy's control scheme

$$\delta x = x_2 - x_1$$

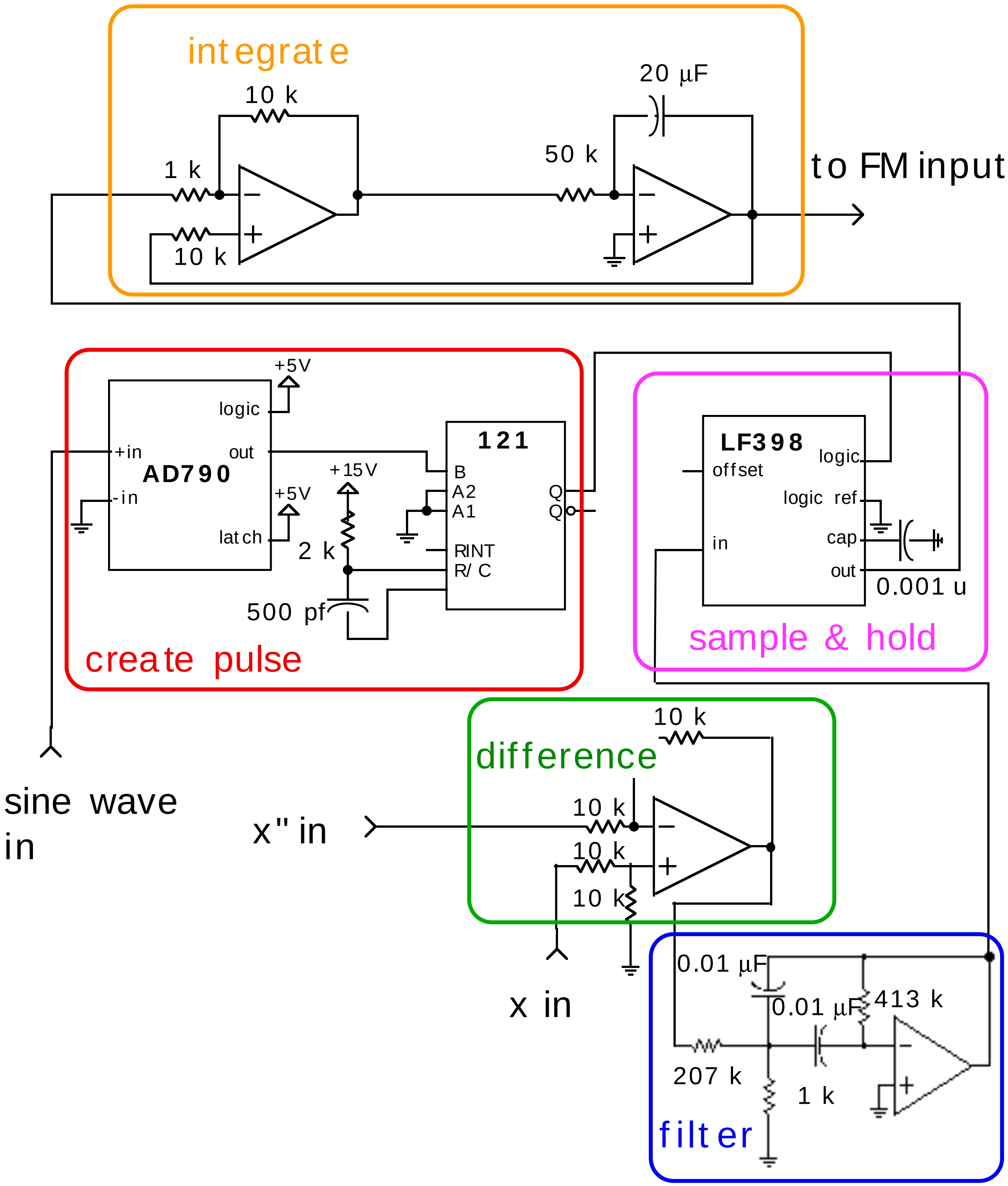
$$\delta\phi(0) = \phi_2(0) - \phi_1(0),$$

It can be shown for small δx after an initial transient time that:

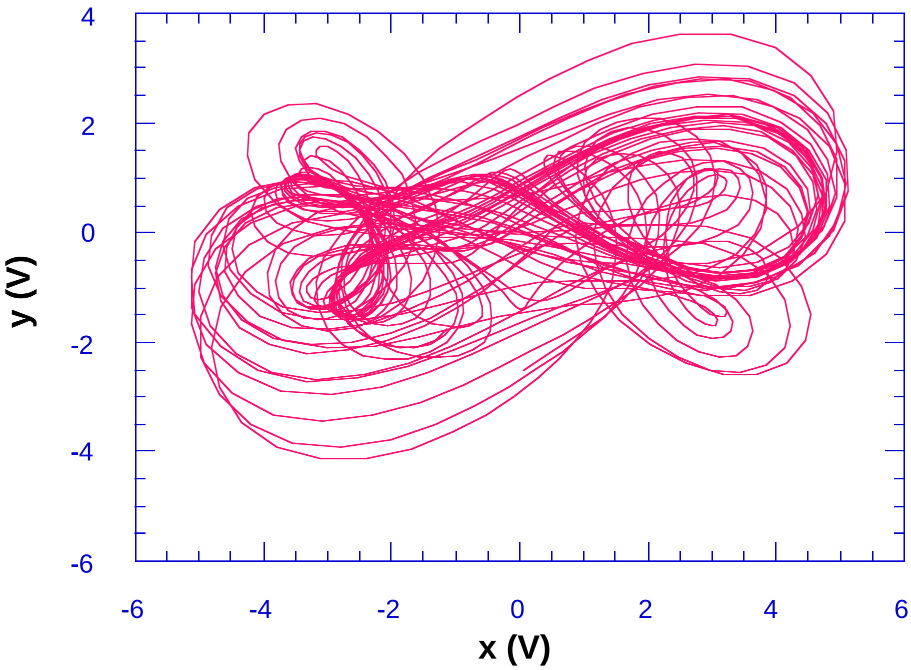
$$\delta x = \frac{A\delta\phi(0)}{\omega(\gamma^2 + \omega^2)}\left[\gamma\cos(\omega t + \phi_2(0)) + \omega\sin(\omega t + \phi_2(0))\right]$$

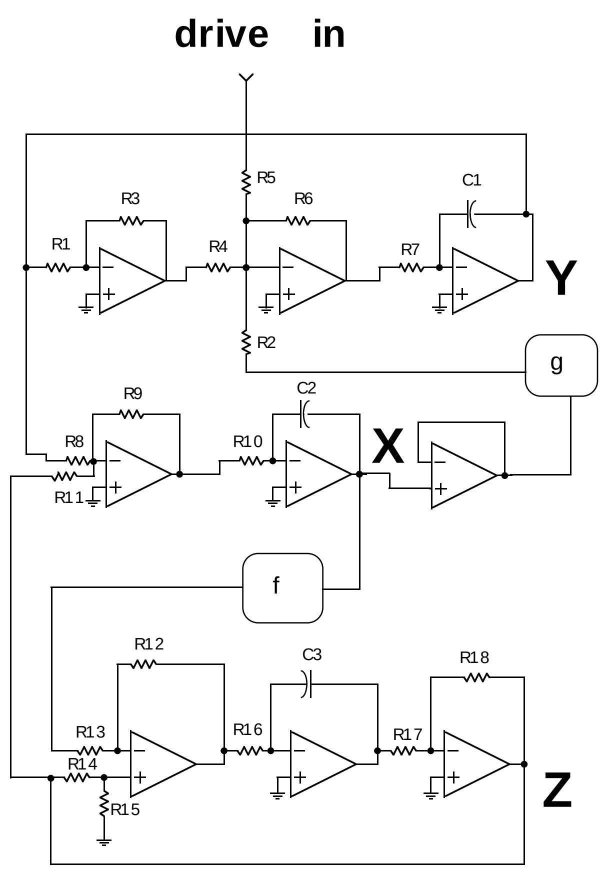
Therefore, the amplitude of δx is proportional to the phase difference between the two systems and is modulated at the forcing frequency ω.

Phase correction circuit

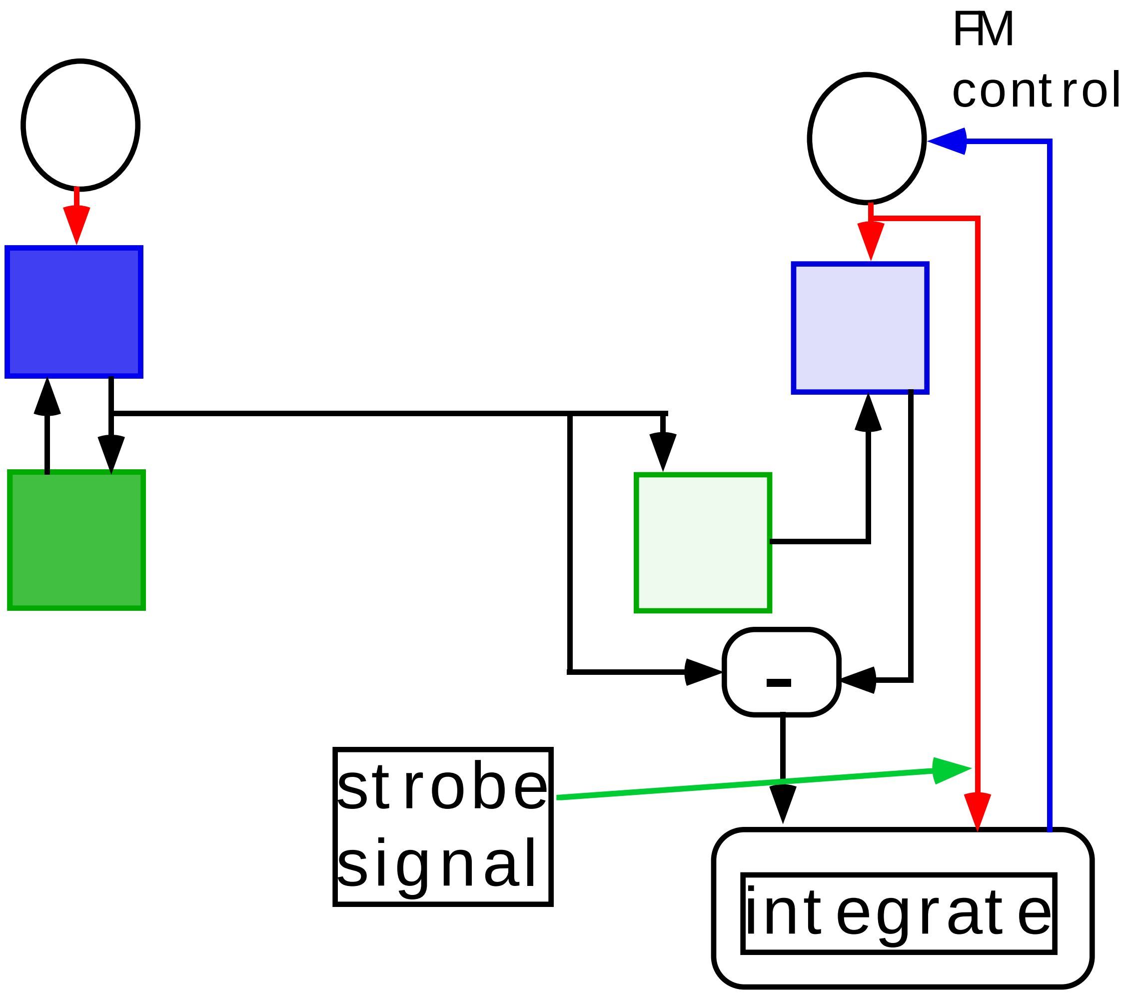


4-D Duffing Circuit

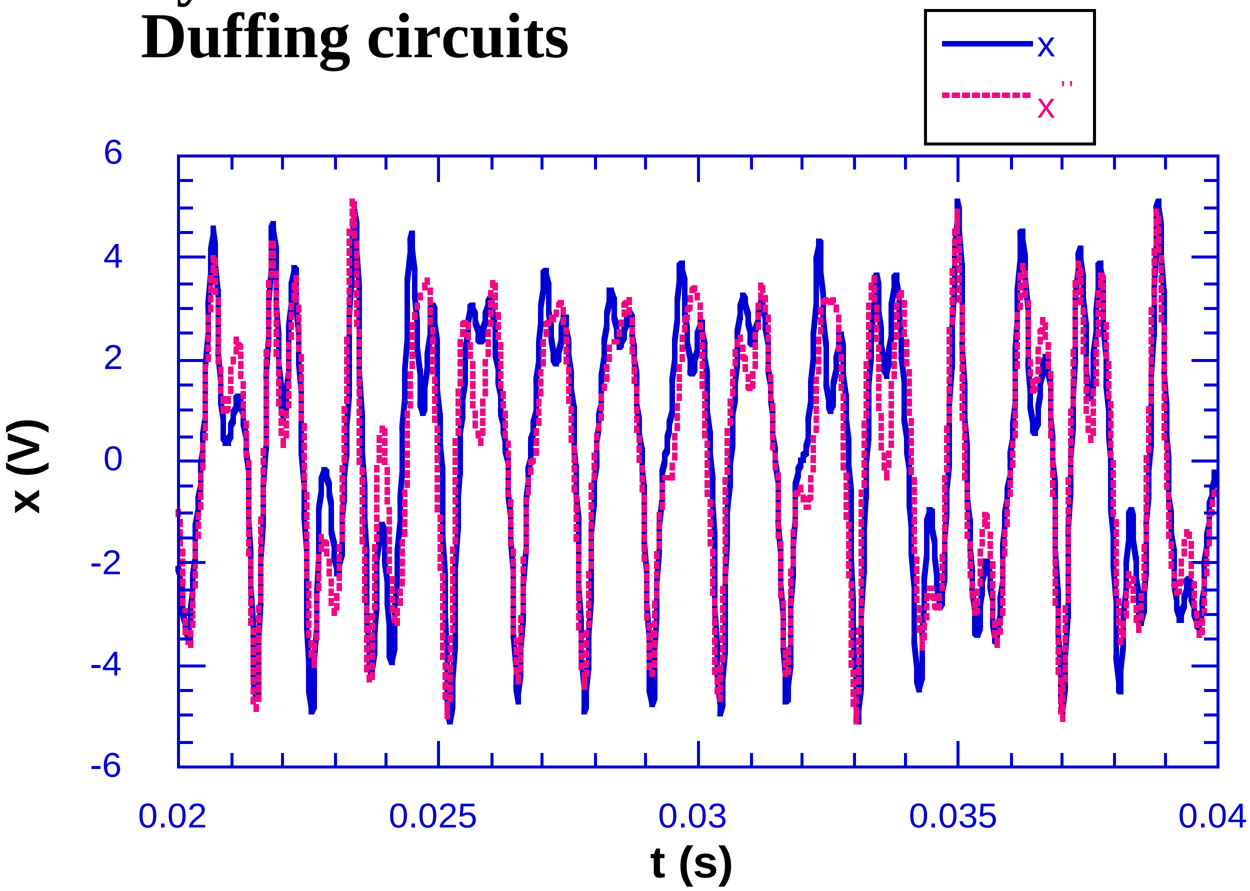


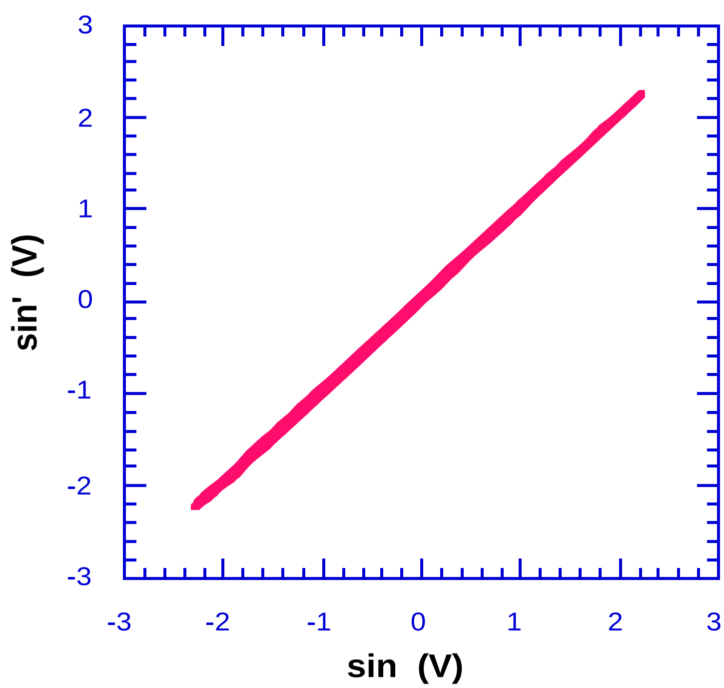


Synchronizing nonauto circuits

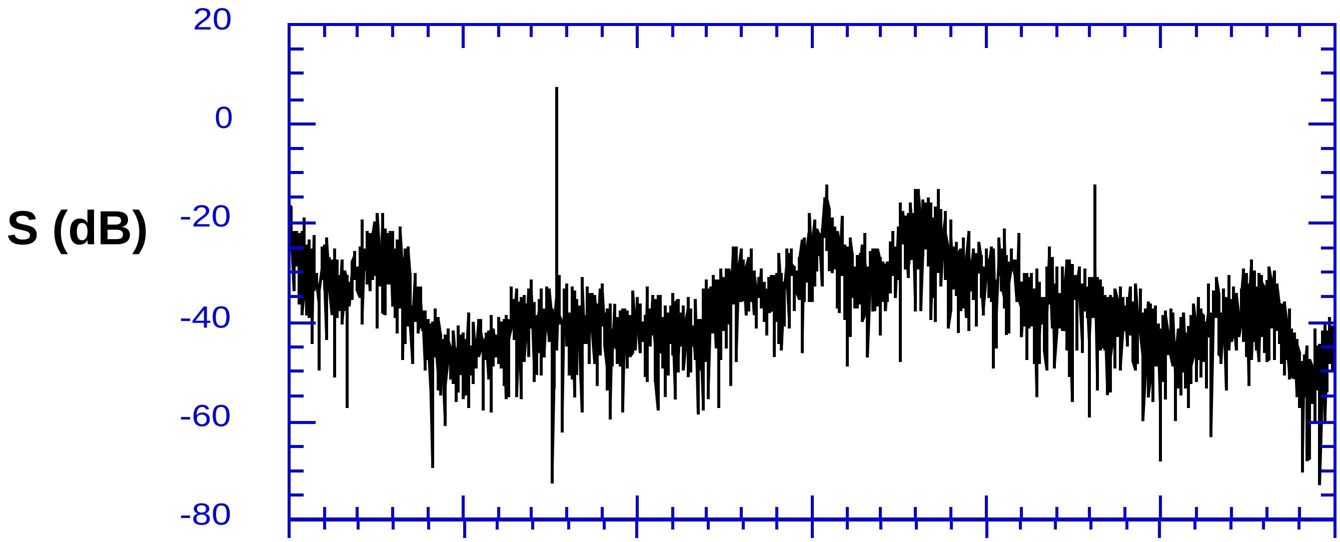


Synchronization of 4-D Duffing circuits

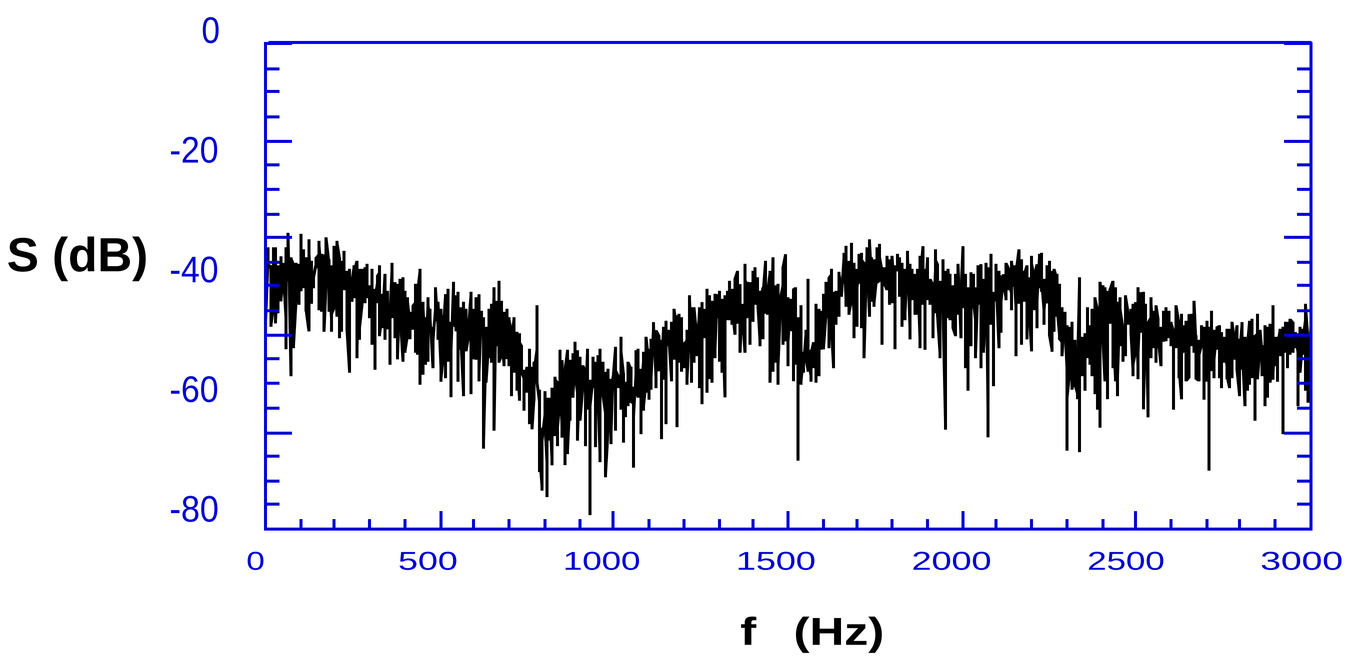




Duffing spectrum unfiltered

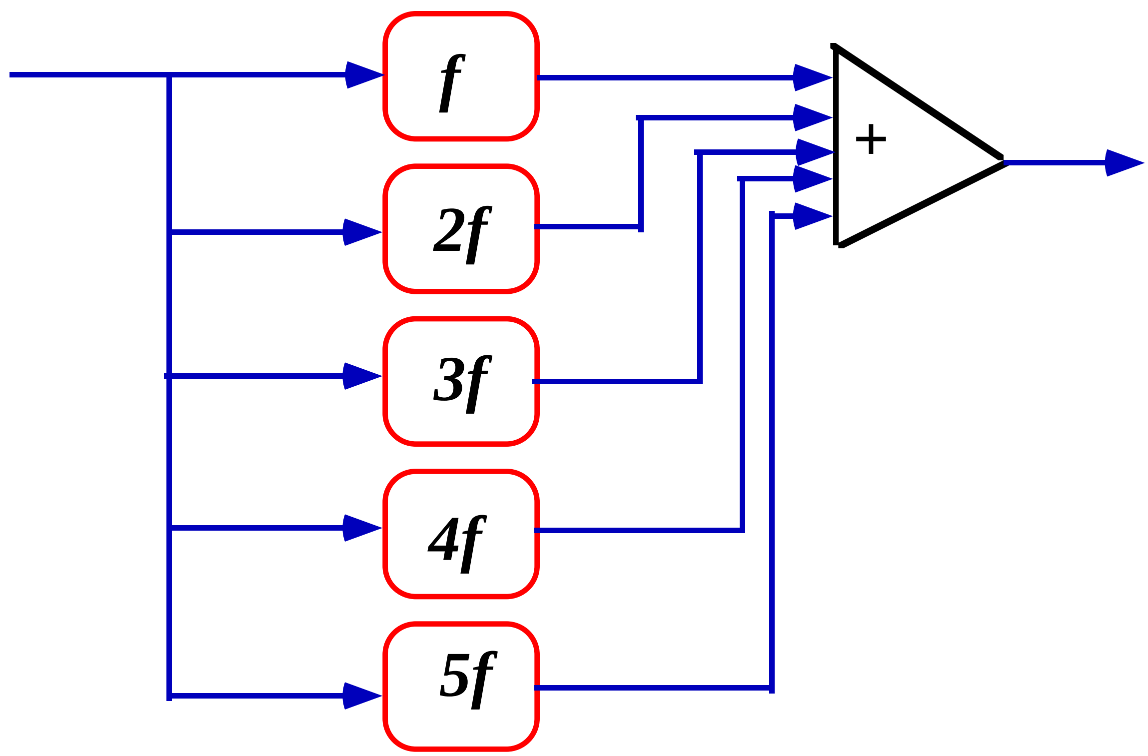


Duffing spectrum filtered

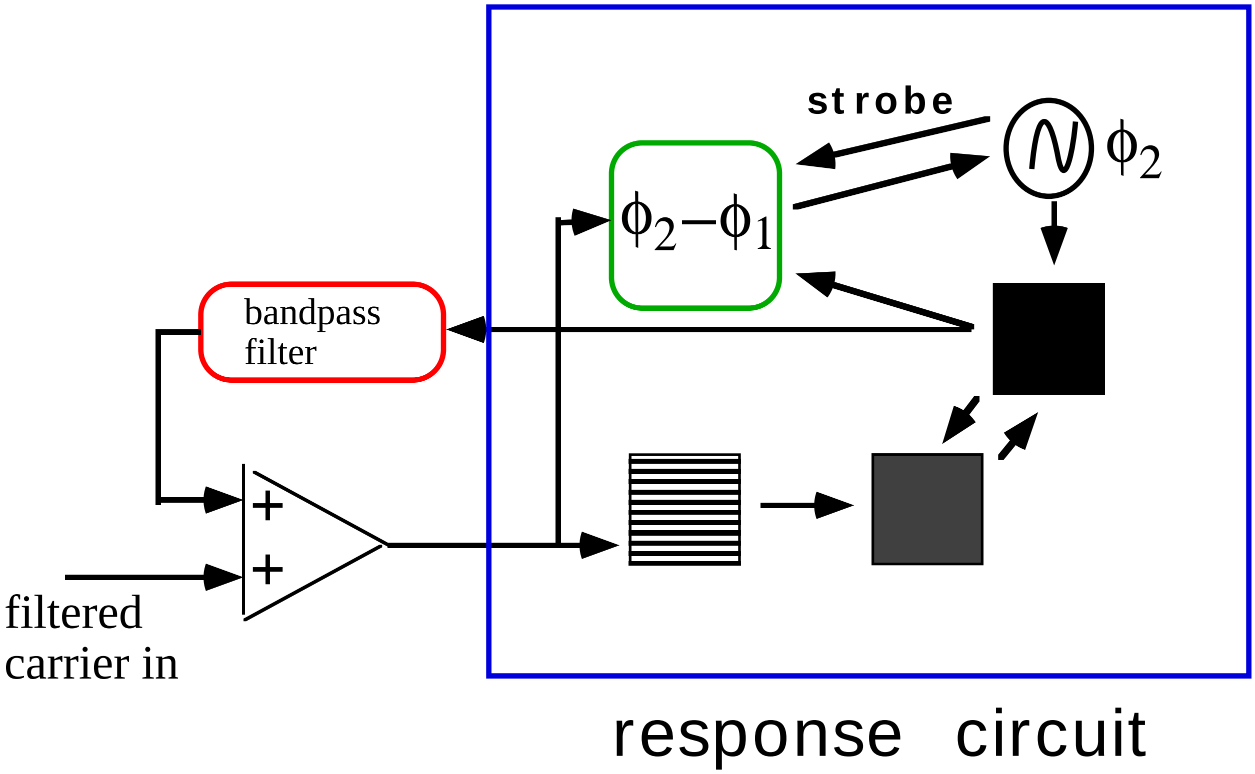


Filtering Duffing Signal

inside bandpass filter:



Reconstructing Chaotic Signal



Full circuit equations

$$\frac{dx}{dt} = \beta(y - z)$$

$$\frac{dy}{dt} = \beta(-\Gamma_y y - g(x) + \alpha\cos(\omega t) + A)$$

$$\frac{dz}{dt} = \beta(f(x) - \Gamma_z z)$$

drive

$$\frac{du_i}{dt} = \frac{-2.0}{R_{i2}C}u_i - \frac{1}{R_{i2}C}\left(\frac{1}{R_{i3}C} + \frac{1}{R_{i1}C}\right)v_i - \frac{1}{R_{i1}C}\frac{dx}{dt}$$

$$\frac{dv_i}{dt} = u_i$$

$$x_t = x + \sum_{i=1}^{5} v_i$$

filter

<u>**x_t is transmitted carrier signal**</u>

$$x_d = x_t - \sum_{i=1}^{5} r_i$$

$$\frac{dq_i}{dt} = \frac{-2.0}{R_{i2}C}q_i - \frac{1}{R_{i2}C}\left(\frac{1}{R_{i3}C} + \frac{1}{R_{i1}C}\right)r_i - \frac{1}{R_{i1}C}\frac{dx''}{dt}$$

$$\frac{dr_i}{dt} = q_i$$

filter

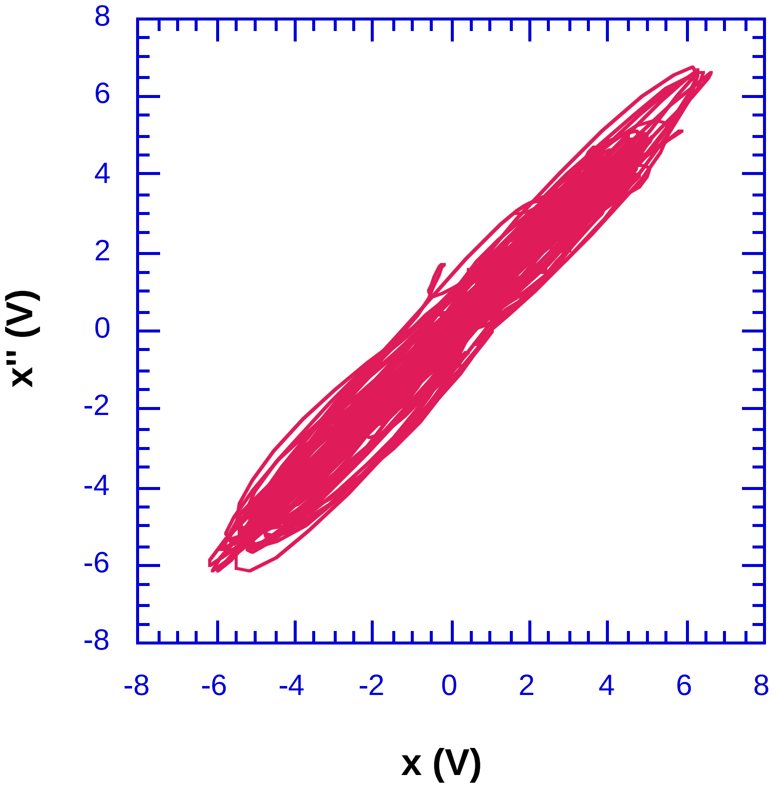
$$\frac{dz'}{dt} = \beta(f(x_d) - \Gamma_z z')$$

$$\frac{dx''}{dt} = \beta(y'' - z')$$

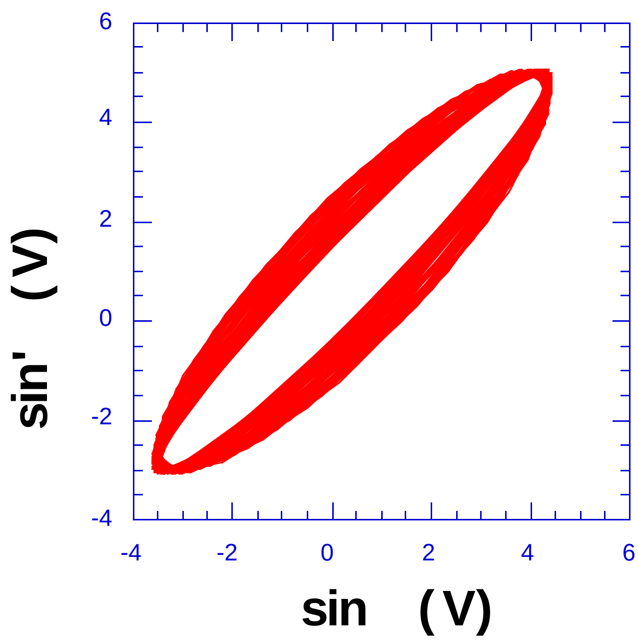
$$\frac{dy''}{dt} = \beta(-\Gamma_y y'' - g(x'') + \alpha\cos(\omega_r t + \phi_r) + A)$$

response

synchronization



Phase synchronization



Signal Separation

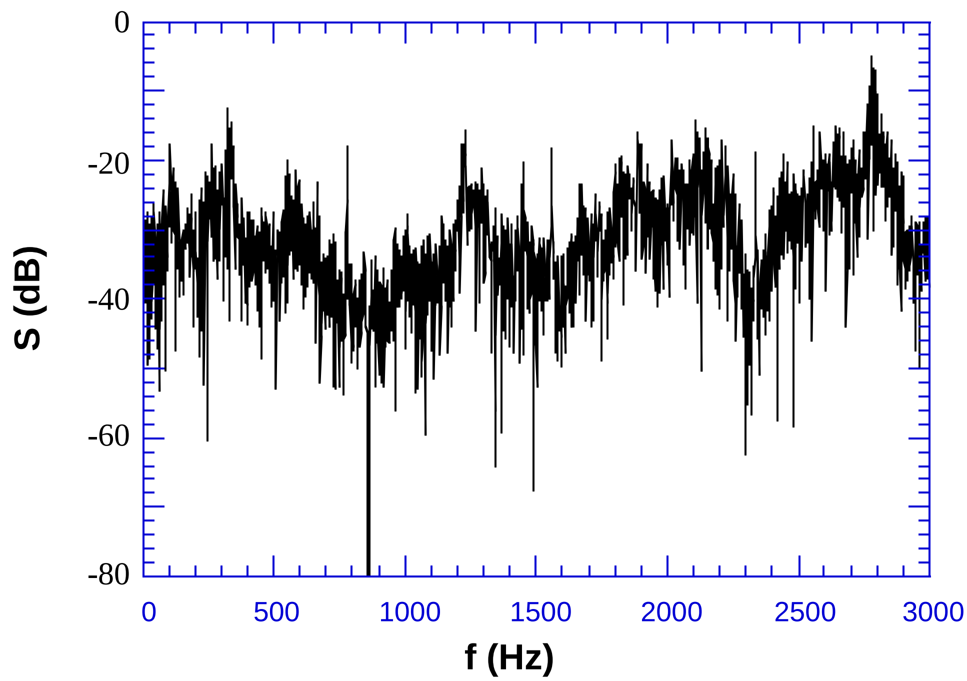
add signal from a different Duffing circuit

Same drive frequency: independent phase

$$\frac{d\xi}{dt} = 10^4\psi$$

$$\frac{d\psi}{dt} = 10^4\left(\beta\cos(\omega t + \phi_2) + A_2 - 0.256\psi - \xi^3\right)$$

filter ξ to produce ξ_t and add to x_t

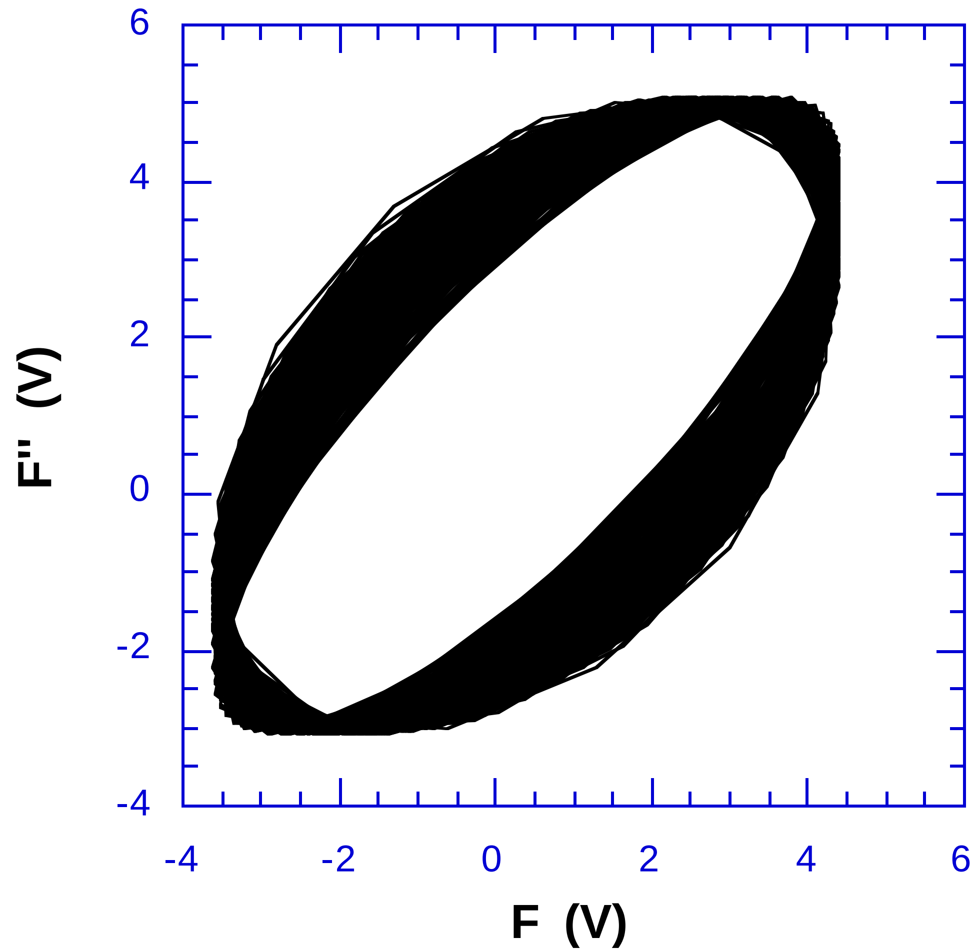


ξ_t power spectrum

ξ_t acts as deterministic in band noise: hard to separate

Signal to noise 1:1

Constant phase offset, but little drift



If only wrong signal present, no synchronization

Review

Drive-Response synchronization:

$$\frac{dx}{dt} = F(x,y,z), \quad \frac{dy}{dt} = G(x,y,z), \quad \frac{dz}{dt} = H(x,y,z)$$

response:

$$\frac{dy'}{dt} = G(x,y',z'), \quad \frac{dz'}{dt} = H(x,y',z')$$

if all Lyapunov exponents <0, *y'*->*y*, *z'* -> *z*

Cascading:

$$\frac{dx''}{dt} = F(x'',y',z''), \qquad \frac{dz''}{dt} = H(x'',y',z'')$$

If all Lyapunov exponents < 0, then *x''* -> *x*

Can communicate by varying a parameter, so *x''* = *x* or *x''* ≠ *x*

Review

Synchronous substitution:

$$\frac{dx}{dt} = F(x, y, z), \quad \frac{dy}{dt} = G(x, y, z), \quad \frac{dz}{dt} = H(x, y, z)$$

transmit *w* = T(*x*,*y*,*z*)

at response: use response variables to invert T:

$$\tilde{x} = T^{-1}(w, y', z')$$

$$\frac{dx'}{dt} = F(\tilde{x}, y', z'), \quad \frac{dy'}{dt} = G(\tilde{x}, y', z'), \quad \frac{dz'}{dt} = H(\tilde{x}, y', z')$$

Choice of T affects stability of response

Review

General coupling scheme:

$$\frac{dx}{dt} = F(x,y,z), \quad \frac{dy}{dt} = G(x,y,z), \quad \frac{dz}{dt} = H(x,y,z)$$

Transmit $w = k_1 x + k_2 y + k_3 z$

Response: $w' = k_1 x' + k_2 y' + k_3 z'$

$$\frac{dx'}{dt} = F(x', y', z') + b_1(w - w')$$
$$\frac{dy'}{dt} = G(x', y', z') + b_2(w - w')$$
$$\frac{dz'}{dt} = H(x', y', z') + b_3(w - w')$$

Vary *k*'s, *b*'s to minimize response Lyapunov exponents.

First 2 schemes are special cases of this general coupling.

Review

Filtering autonomous chaotic systems:

$$w = k_1 x + k_2 y + k_3 z$$

Transmit $w_f = \Phi(w)$**, where** Φ **is a dynamical system (filter).**

At response, $w' = k_1 x' + k_2 y' + k_3 z'$

$$w'_f = \Phi(w')$$

Feedback $(w_f - w'_f)$ **to synchronize response system.**

Review

Synchronizing non-autonomous systems.

$$\frac{d\vec{x}}{dt} = F\left(x, y, z, \sin\left(\omega t + \phi_1\right)\right)$$

response:

$$\frac{d\vec{x}'}{dt} = F\left(x, y', z', \sin\left(\omega t + \phi_2\right)\right)$$

then ***x-x'*** **is proportional to**

$$(\phi_1 - \phi_2)\sin(\omega t + \theta)$$

Filter at ω, sample, integrate, and correct response phase using frequency modulation.

Review

Filtering nonautonomous signals:

$$\frac{d\vec{x}}{dt} = F\left(x, y, z, \sin\left(\omega t + \phi_1\right)\right)$$

Filter out periodic components of transmitted signal:

$x_f = x - \Phi(x)$**, where** Φ **is a bandpass filter**

At response, $x'_\phi = \Phi(x')$

Drive response with

$$\tilde{x} = x_f + x'_\phi = x - \Phi(x) + \Phi(x')$$

Result is phase sync of sine oscillators by broad-band signal. Very noise resistant.

Non-self-synchronous systems

• Problems with self-synchronous systems:

noise added to reference signal- is it possible to use when S/N< 1? (no processing gain)

Distortion: filtering may be irreversible, information lost.

Stable response system necessary: difficult when filters added.

Circuits must be well matched.

Extracting signals from noise

Must know something about signal. Examples:

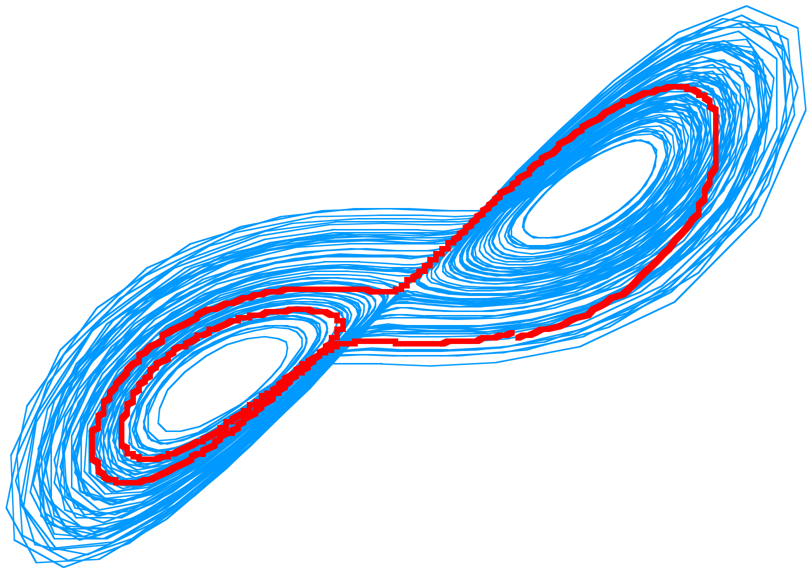
• Periodic carrier- know frequency

• Direct sequence spread spectrum- know pseudo-random sequence.

• Deterministic signals: presence of determinsm used in some chaotic noise reduction methods.

For a known chaotic attractor, we do know the unstable periodic orbits (UPO's)

Unstable periodic orbits



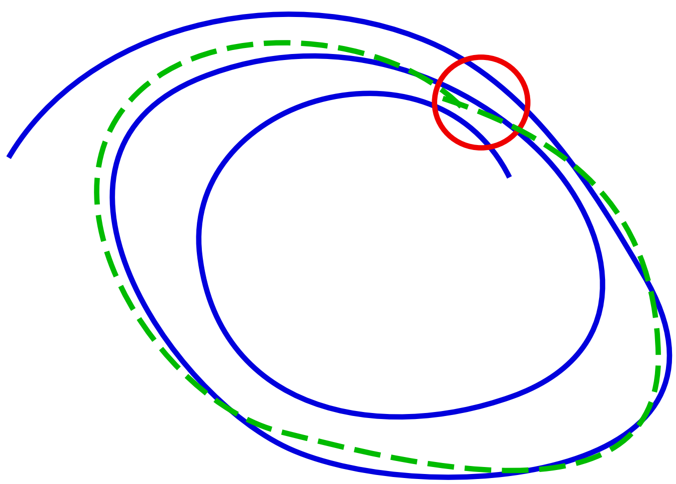
A chaotic attractor consists of an infinite number of UPO's.

If we know which UPO attractor is on at all times, we know attractor.

Finding UPO's

From equations of motion:
- **Use Newton's method to find periodic solutions.**

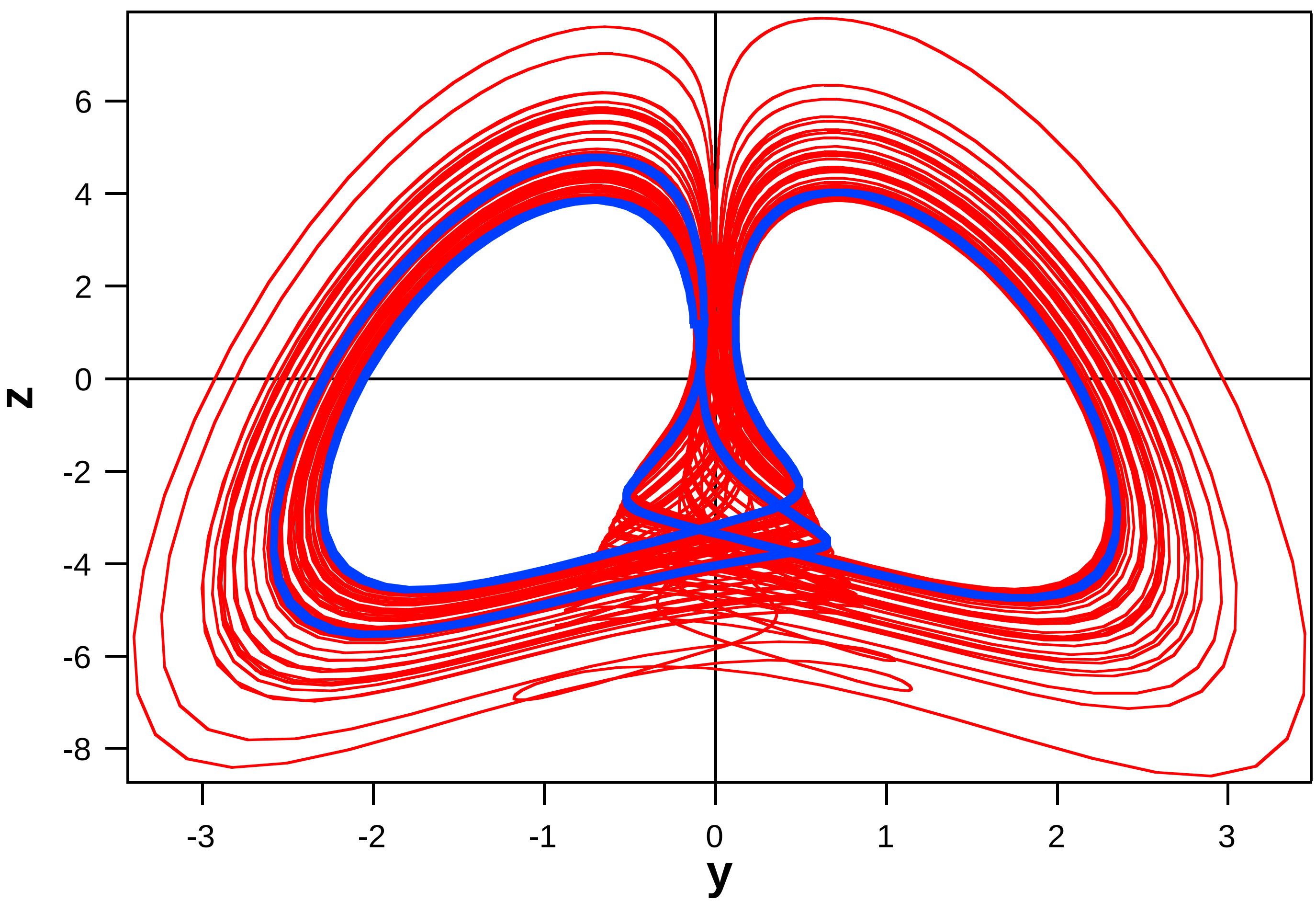
From data:
- **Method of close approaches**



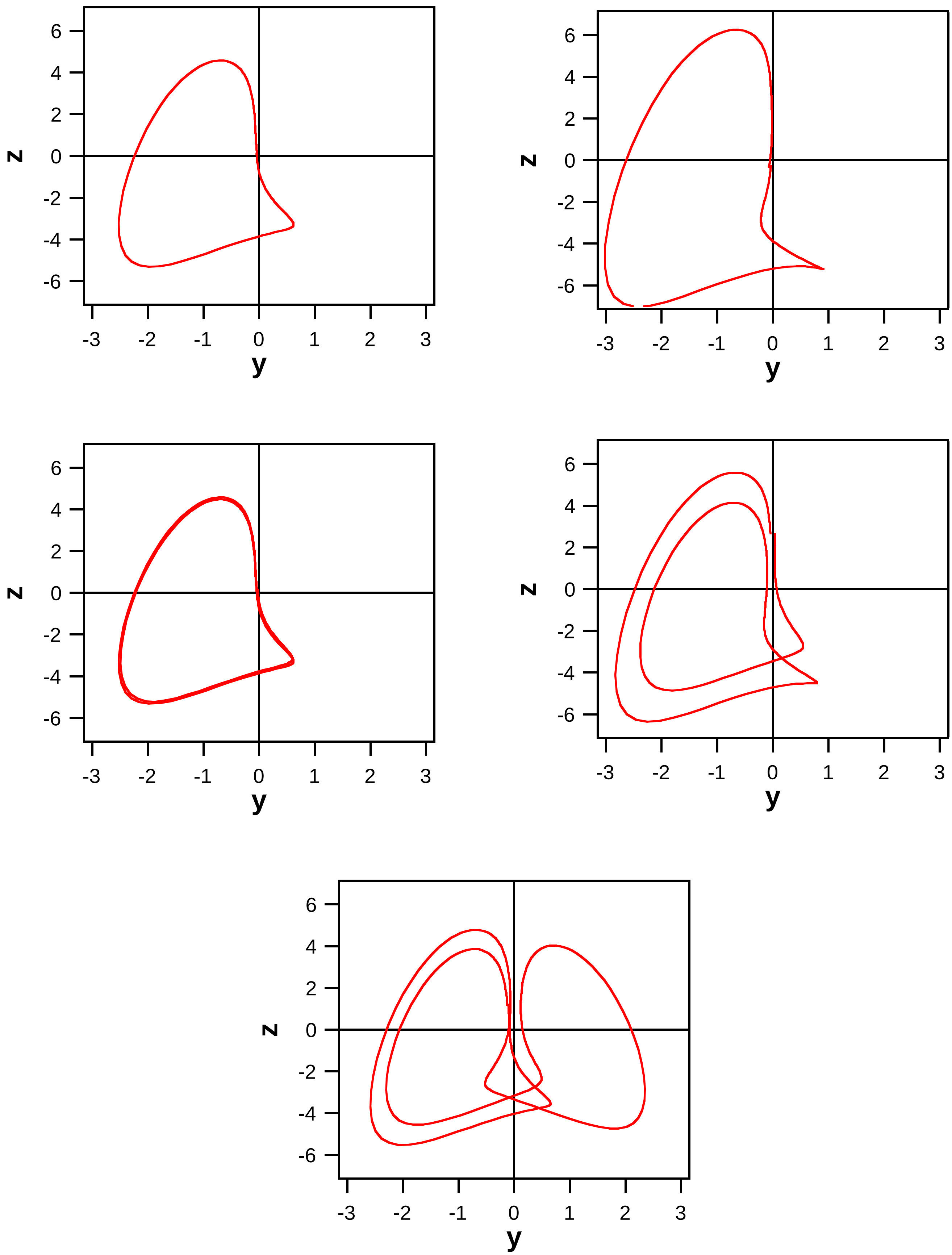
Look for multiple returns within some small radius.

Sprott system B

$$\frac{dx}{dt}=0.4yz$$
$$\frac{dy}{dt}=x-1.2y$$
$$\frac{dz}{dt}=1-xy$$



Typical UPO's from Sprott system B



Unstable periodic orbits

Infinite number of orbits, but dynamics dominated by low period orbits.

Can create an approximation to the attractor by using only a finite set of low period orbits.

UPO Approximation technique

1. **Extract UPO's up to a given length from a chaotic attractor.**

2. **Construct all possible sequences of motion based on this set of UPO's. Truncate sequences to a fixed length L**

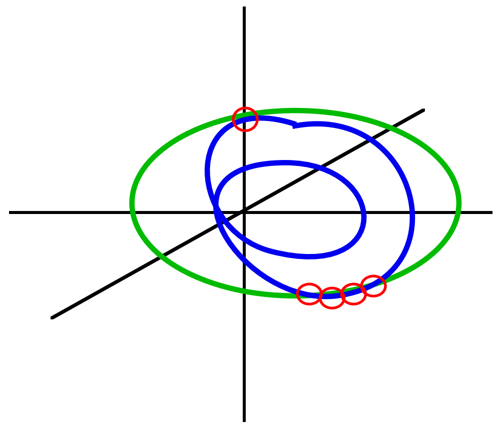
3. **Take segment of length L from time series. Compare to each UPO sequence. Choose best fit to approximate time series.**

4. **Repeat 3**

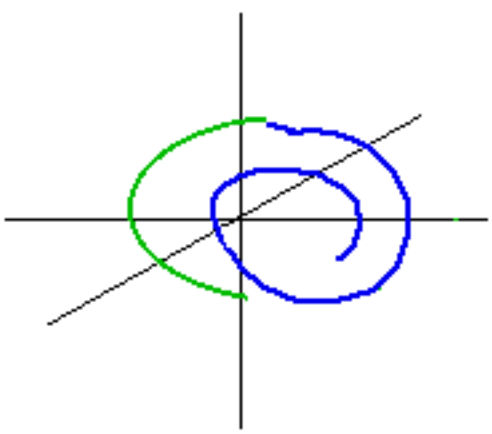
5. **Build histogram, truncate set of UPO's**

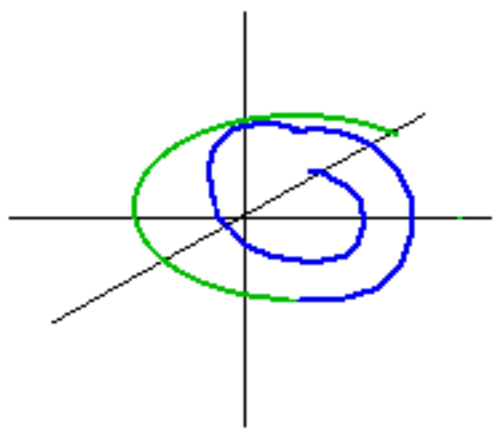
6. **Combine UPO sequences for length of 2L, 3L, etc, if necessary**

Creating UPO sequences



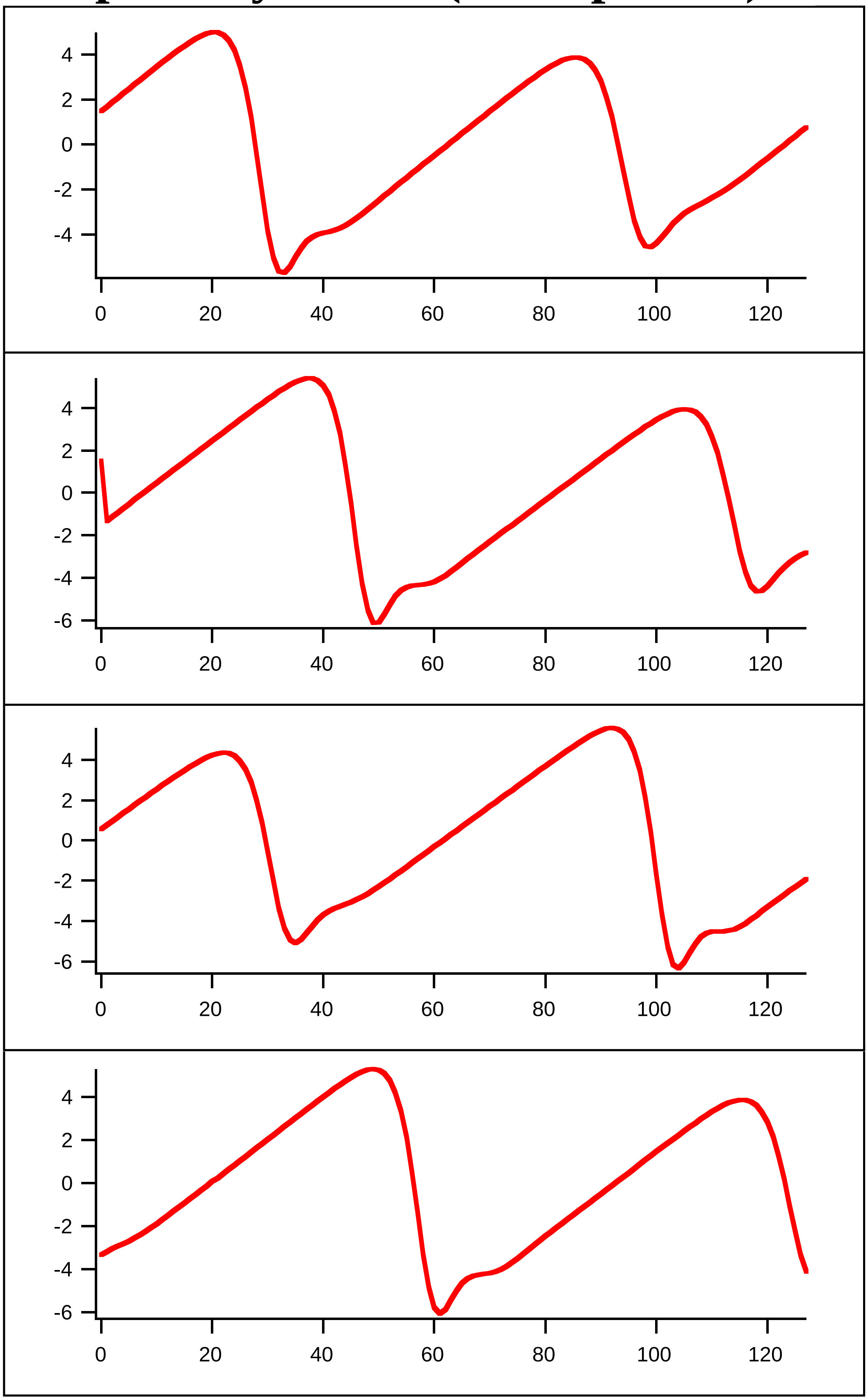
Decide which UPO's can follow each other: x,y, and z must come within some tolorance ε of each other.



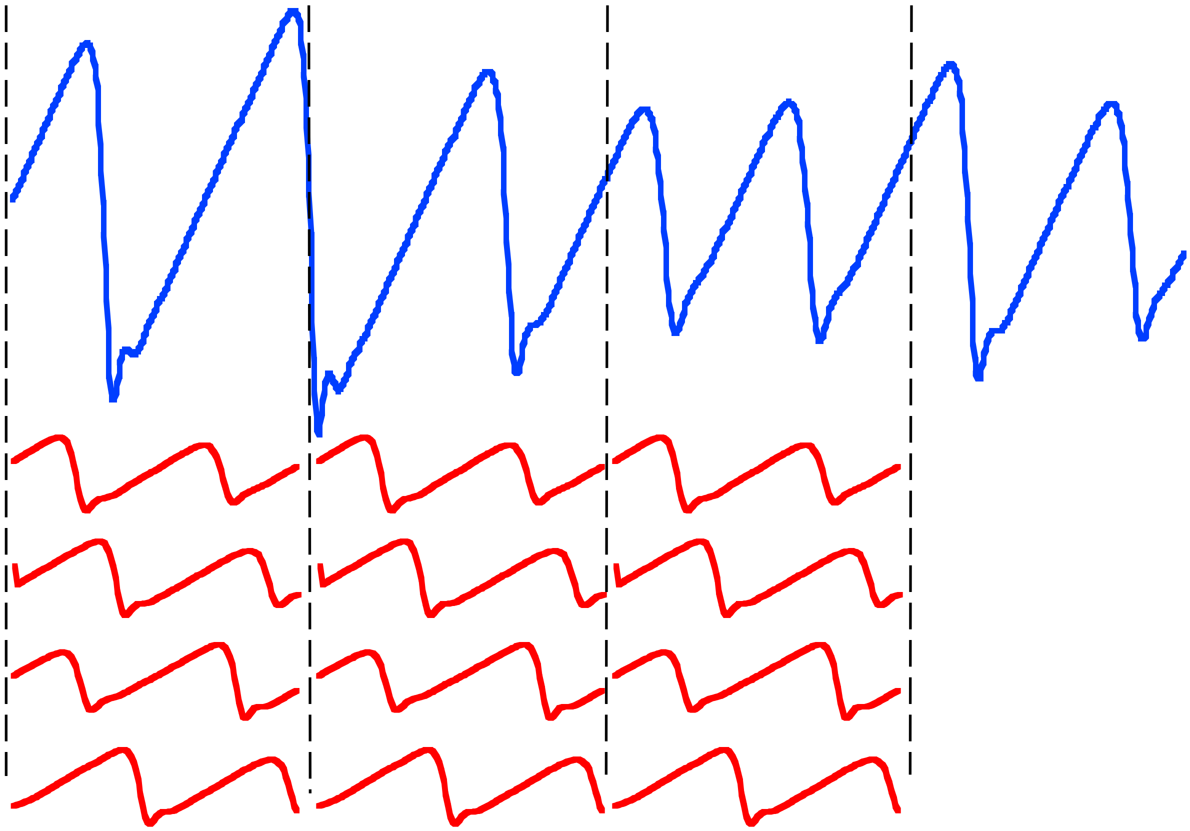


Construct all possible sequences for a fixed length L

Typical UPO sequences of length 128 for Sprott System B (z component)

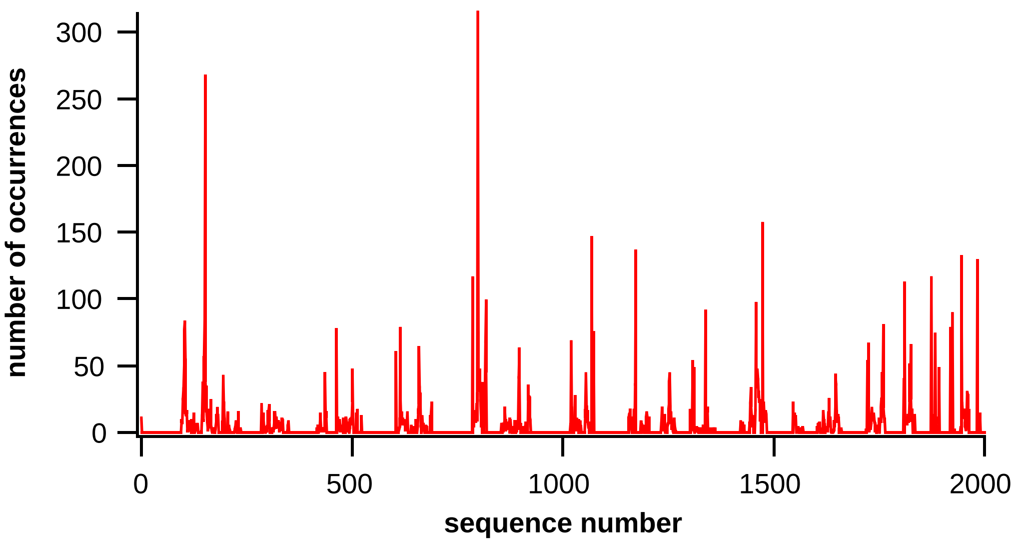


Correlate with incoming signal:

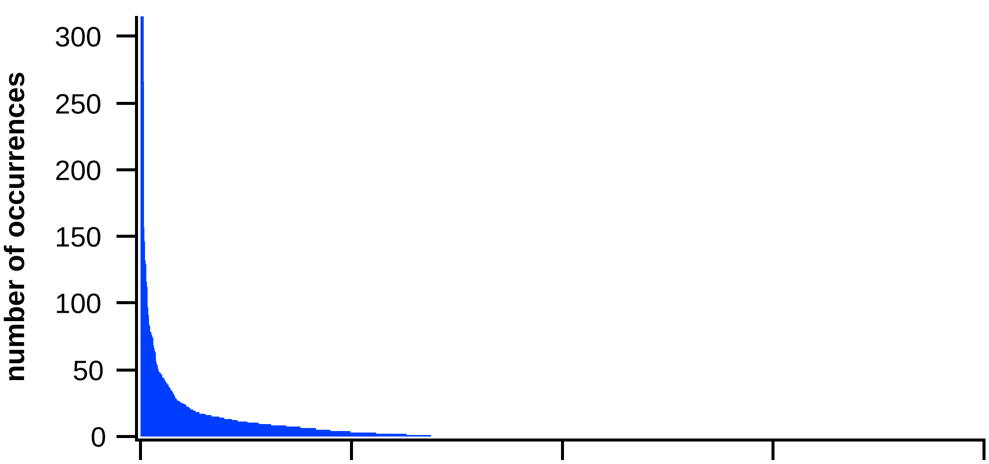


Compare each 128 point sequence. At each interval, choose sequence with largest cross correlation.

Histogram of 128 point sequences (1992 sequences)



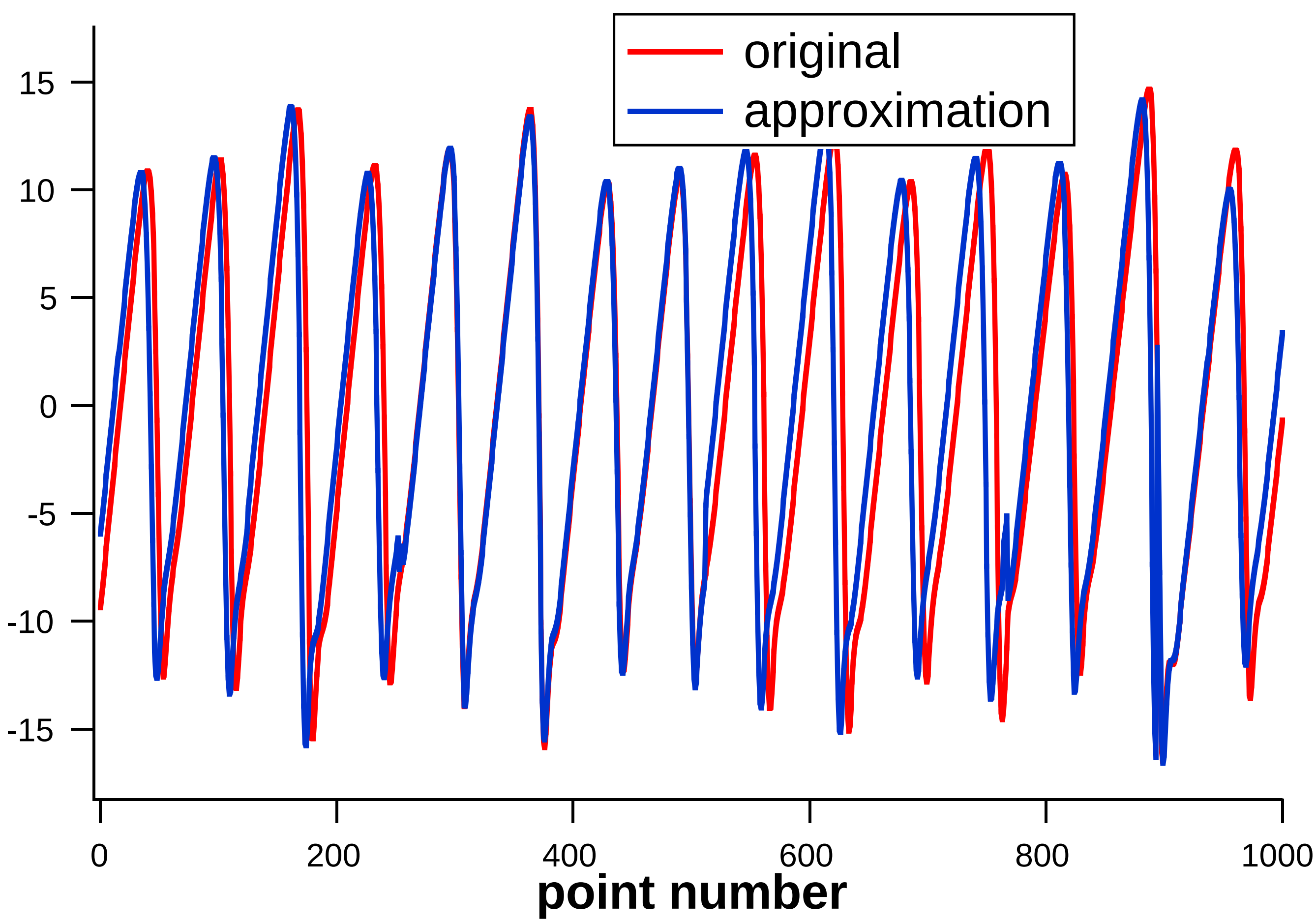
Sorted by number of occurrences



274 sequences occur more than 10 times

combine 128 point sequences for 256 point sequences

keep 1162 sequences

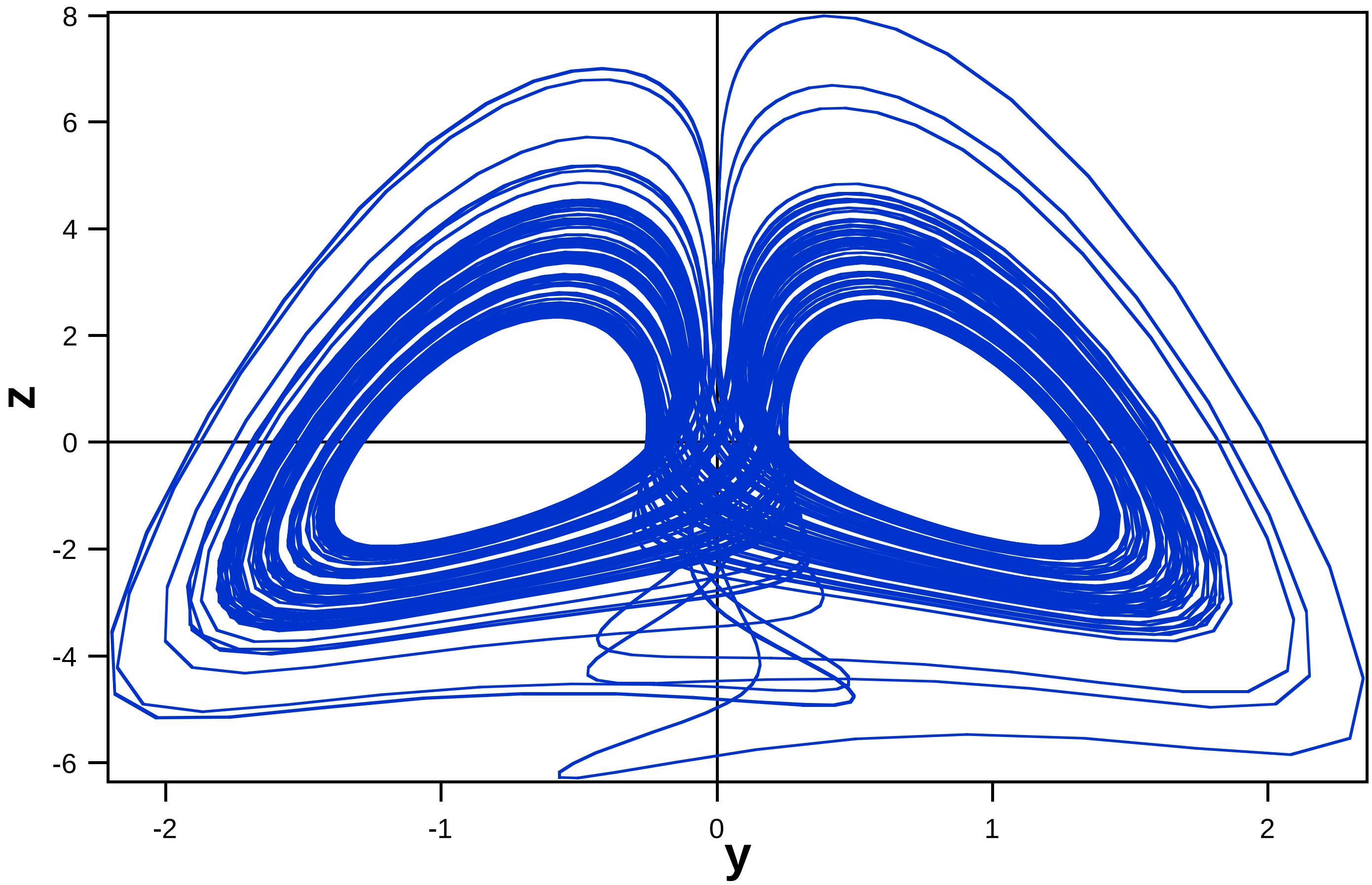


Sprott system C

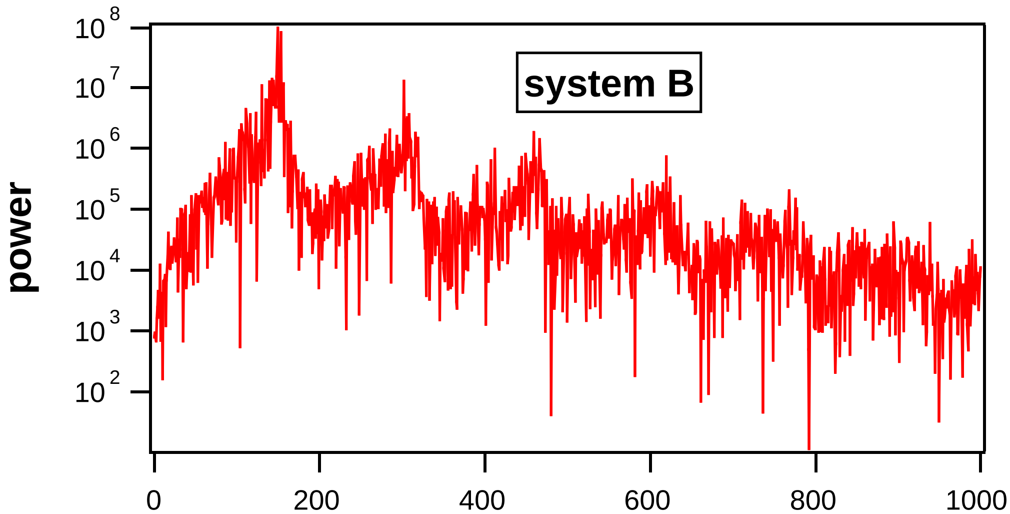
$$\frac{dx}{dt} = 0.4yz$$

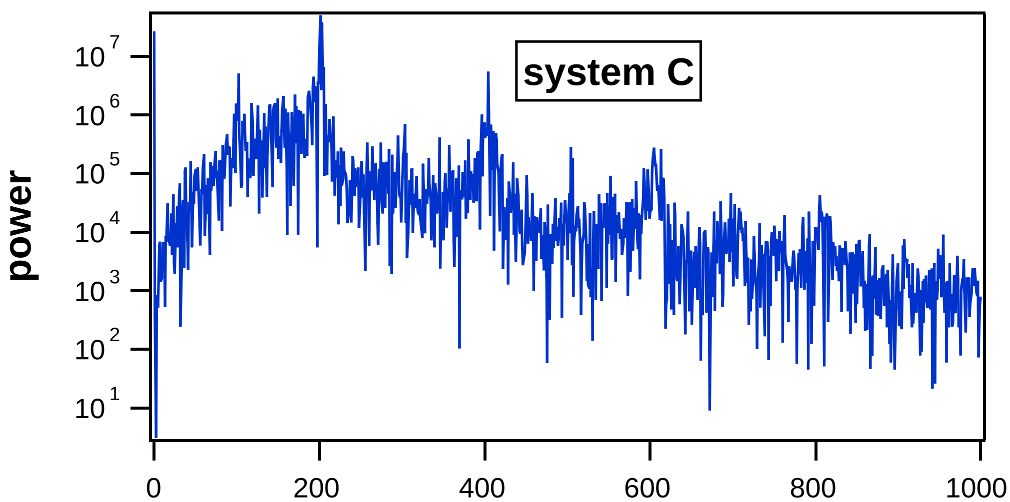
$$\frac{dy}{dt} = x - 1.2y$$

$$\frac{dz}{dt} = 1 - x^2$$

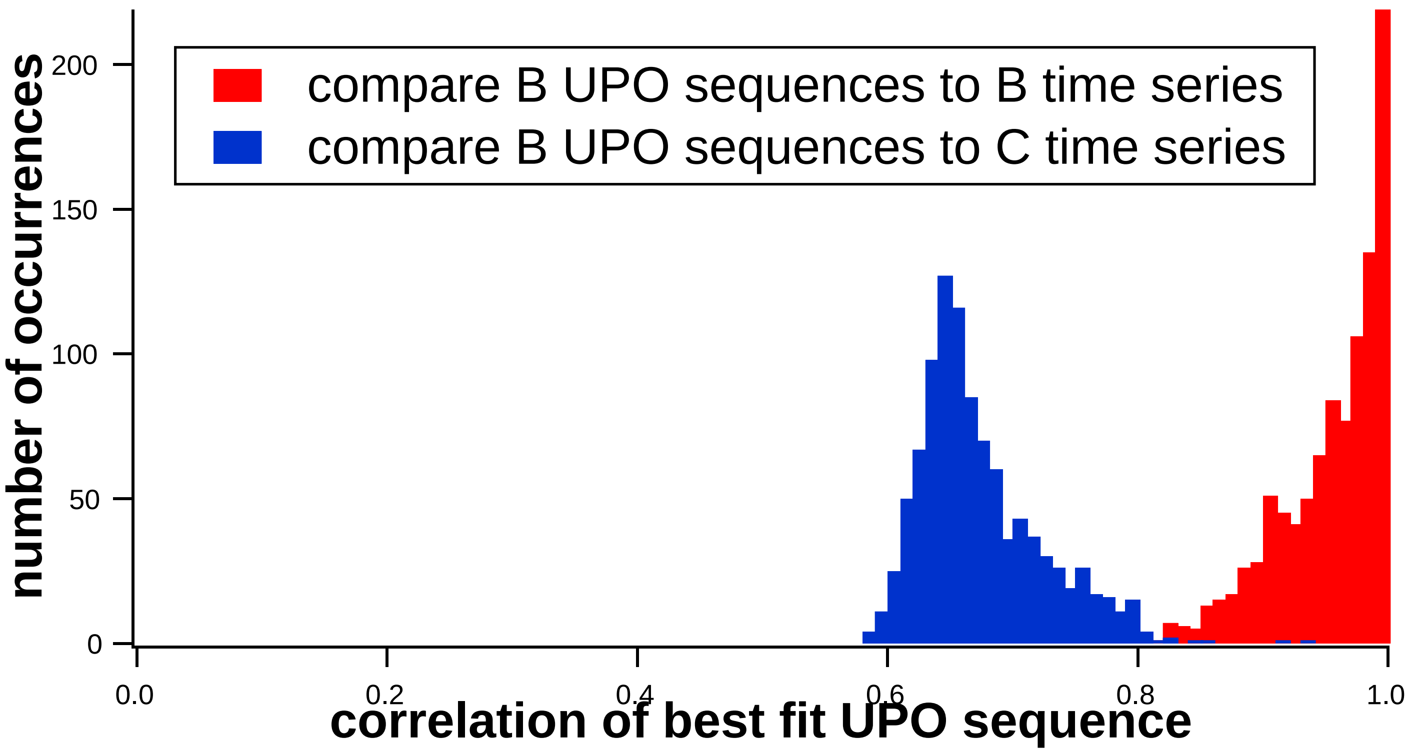


Power spectra for systems B,C
(adjust time constants so spectra match)



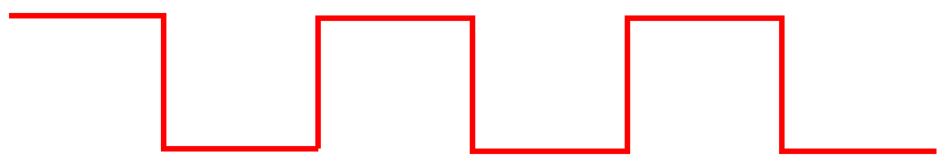


Try to fit system C time series with system B UPO sequences: histogram of correlations.



Communication

- Multiply time series by ± 1



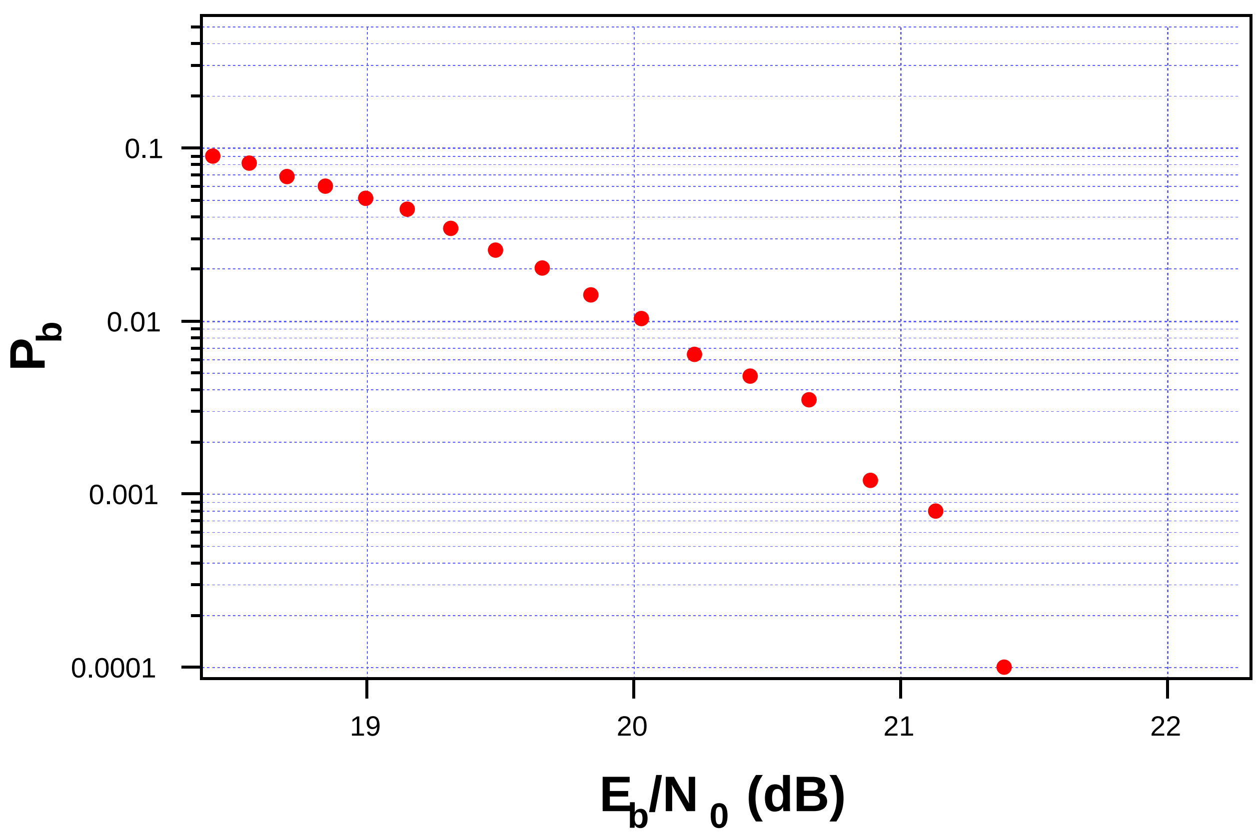
- Add noise or other chaotic signal.

- Use cross-correlation to find best fit UPO sequence: reveals if + or - 1 was sent.

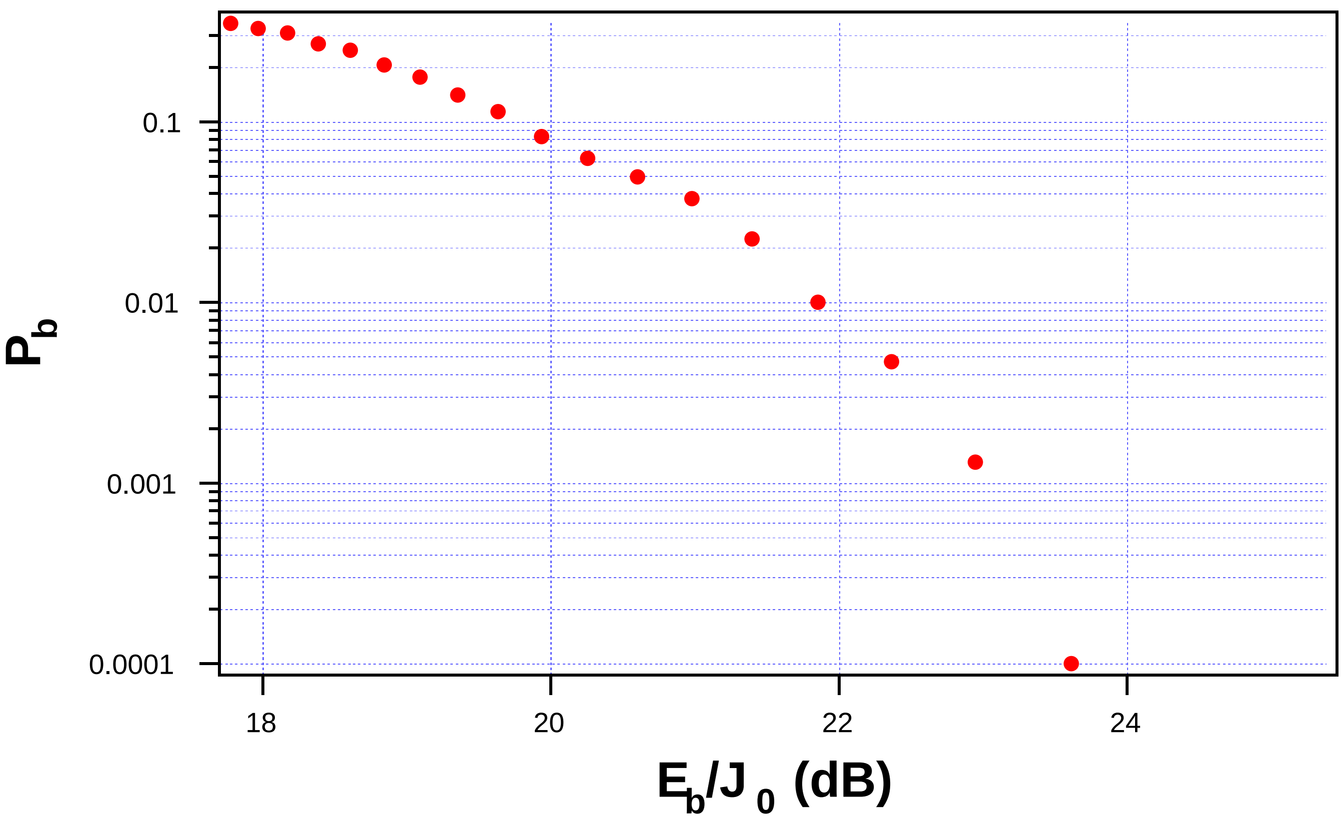
- Estimate bit error probability: what is the probability of detecting +1 when -1 was sent, and vice versa.

- Bit error probability of 0.5 means no knowledge

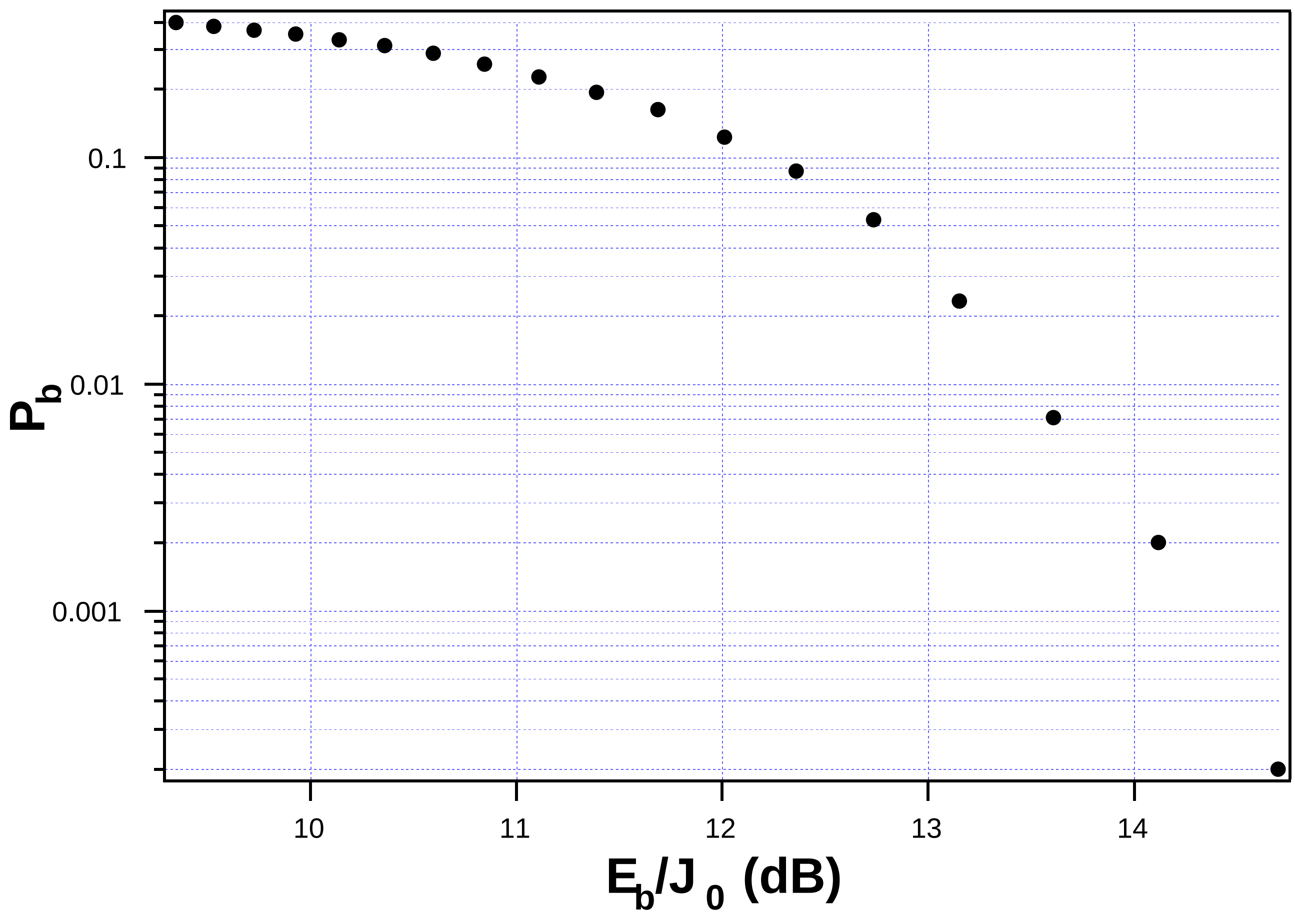
Performance for Sprott system B
with white noise



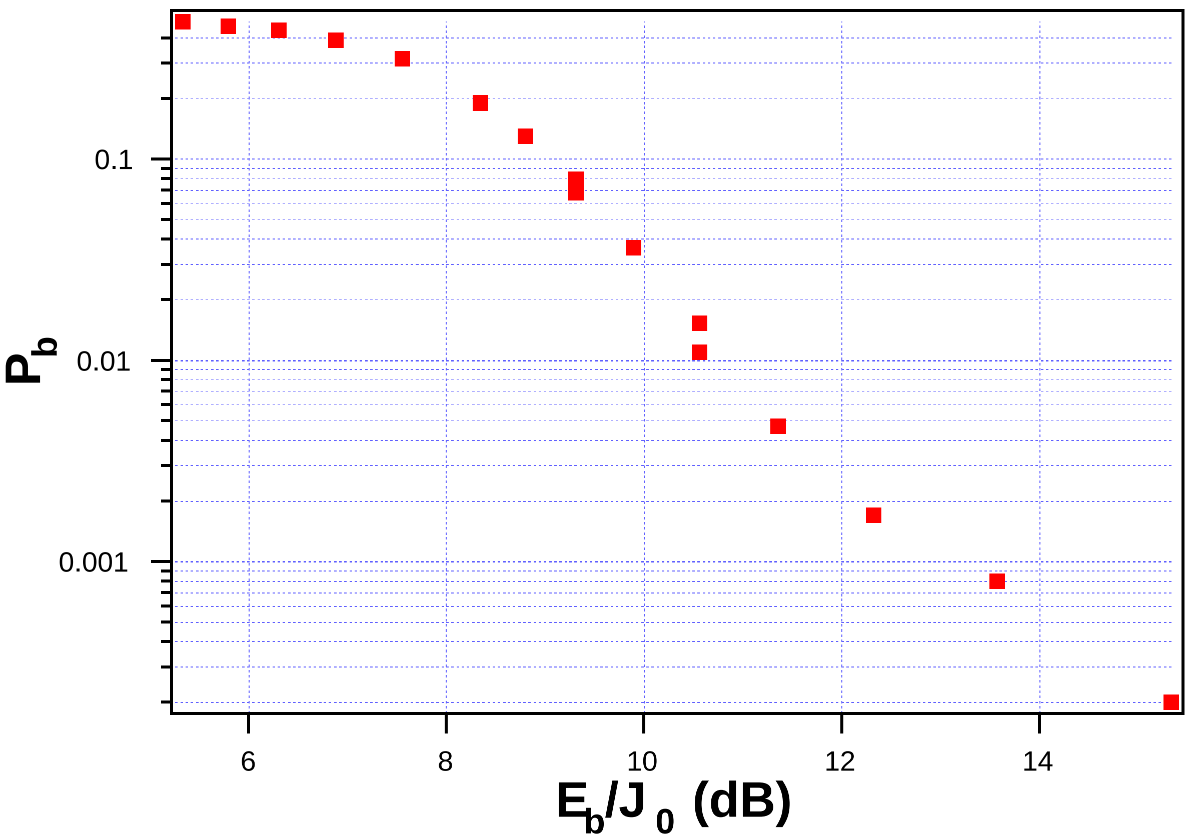
Performance of Sprott system B with Sprott system C



Performance with noise from Lorenz equations



Performance of Logistic map with noise from a 3-D linear map



Using UPO's to overcome distortion

Simulate distortion with a 2nd order band pass filter:

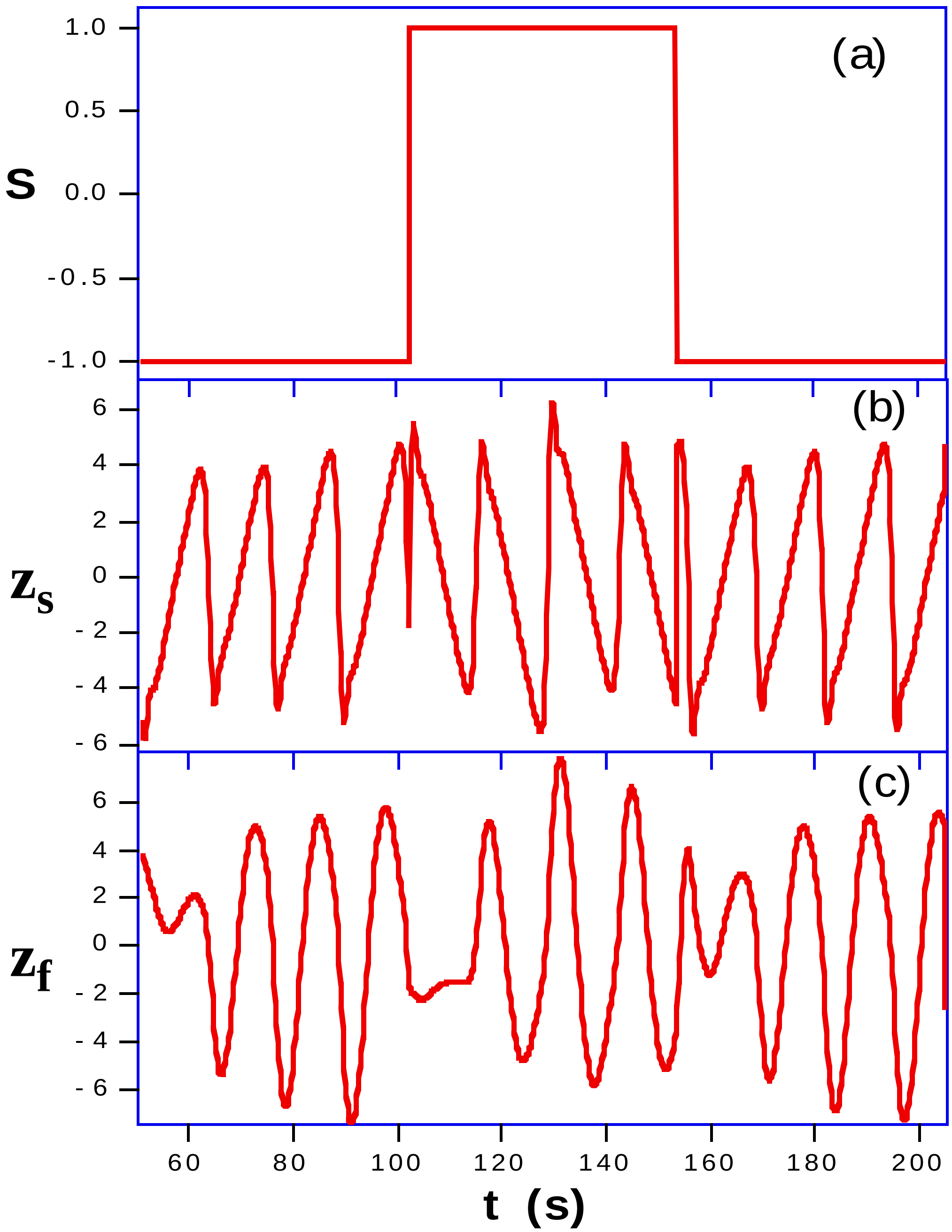
$$A(\omega) = \frac{j\Omega}{Q\left(1 + j\Omega/Q - \Omega^2\right)} \qquad \Omega = \omega/\omega_c$$

Modulate signal from Sprott system B as before.
Filter with bandpass filter.

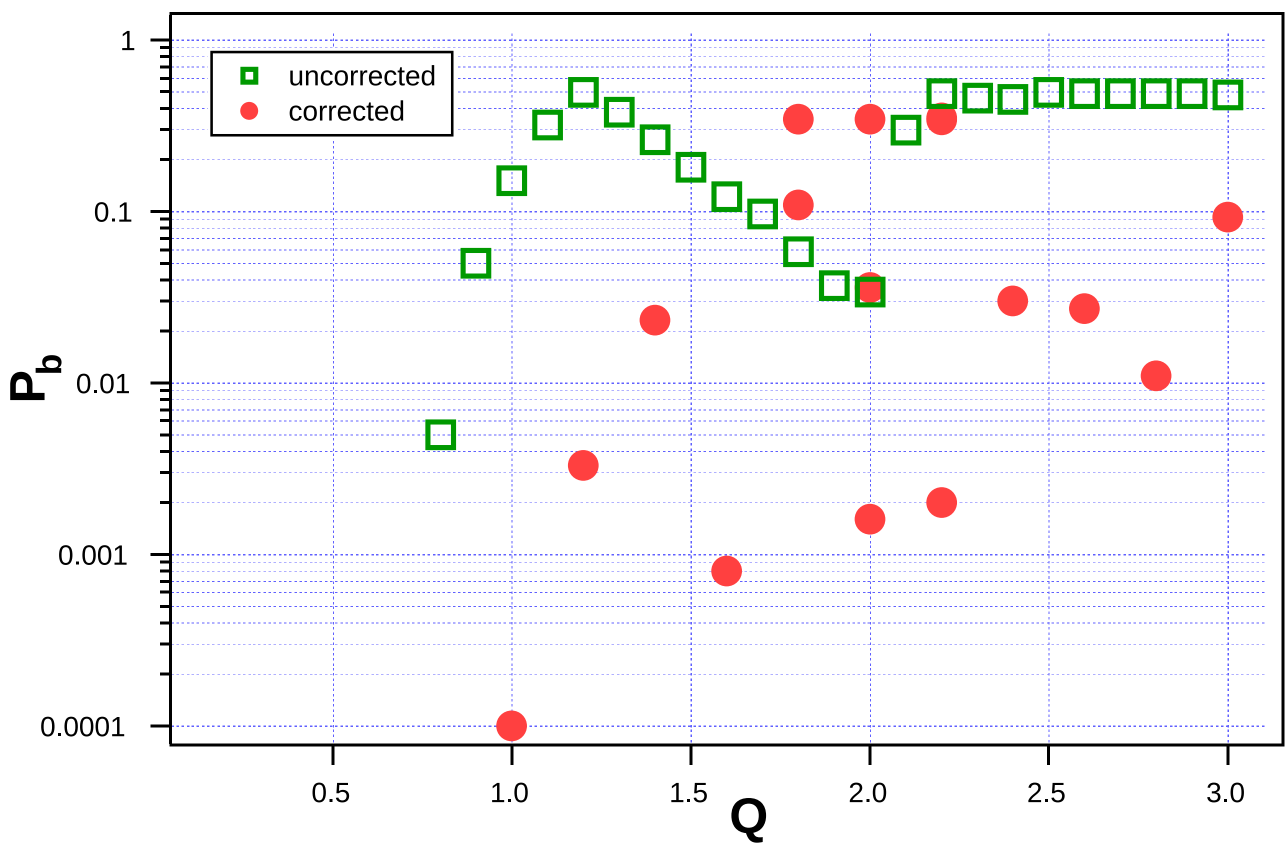
Compare to UPO sequences as before, but first filter with bandpass filter, where ω_c and Q are adjustable parameters.

Vary ω_c and Q during each comparison to get best cross correlation

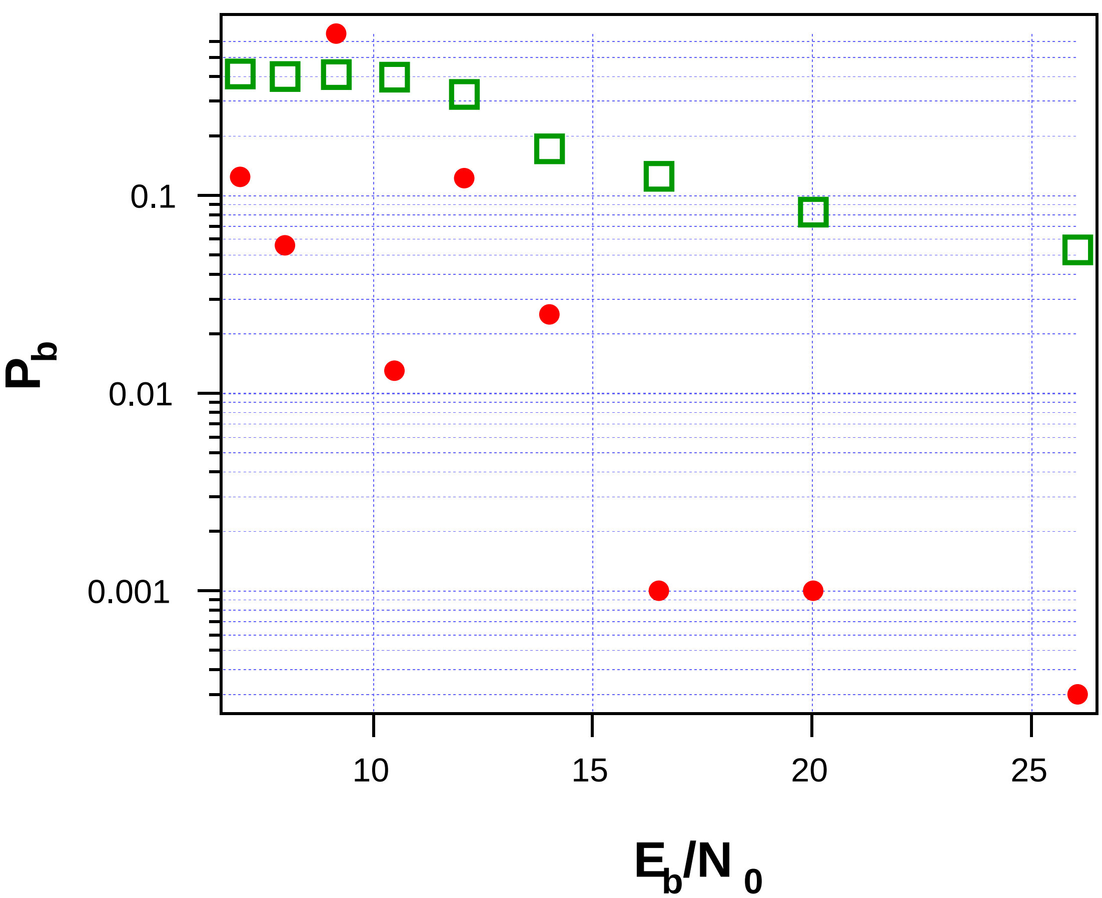
$Q=2$, $\omega_c = 0.5$


(a)
(b)
(c)
s
1.0
0.5
0.0
-0.5
-1.0
z_s
6
4
2
0
-2
-4
-6
z_f
6
4
2
0
-2
-4
-6
60
80
100
120
140
160
180
200
t (s)

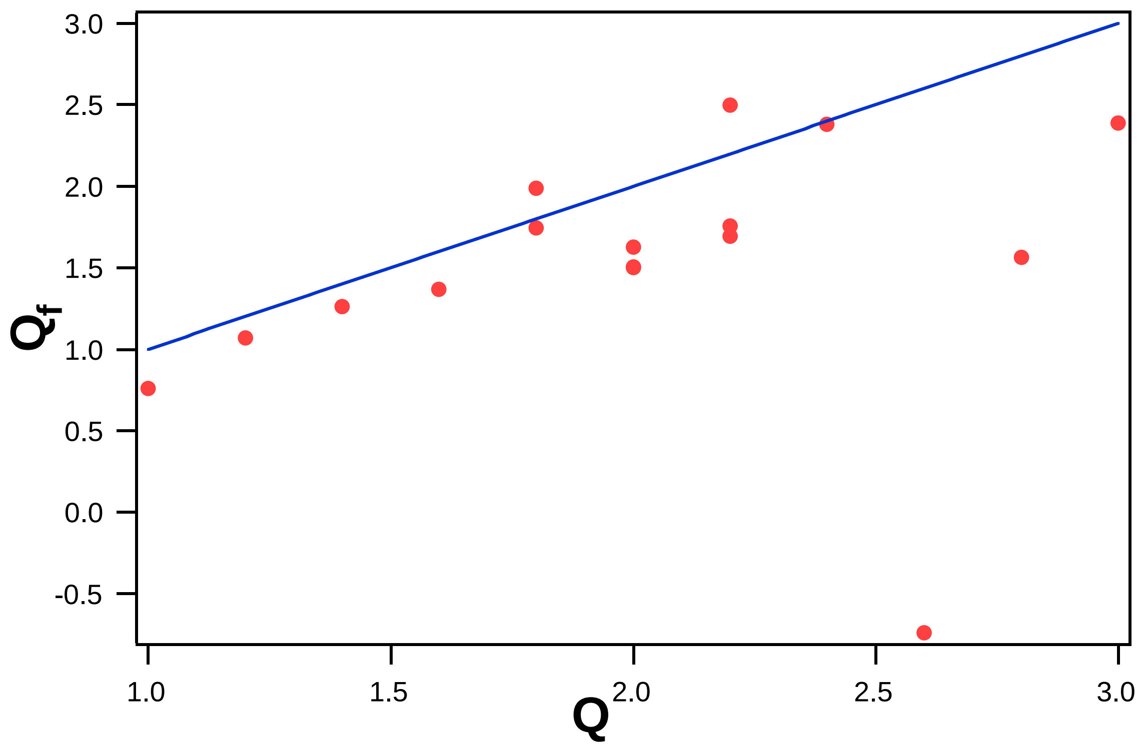
Performance vs filter Q
uncorrected and corrected



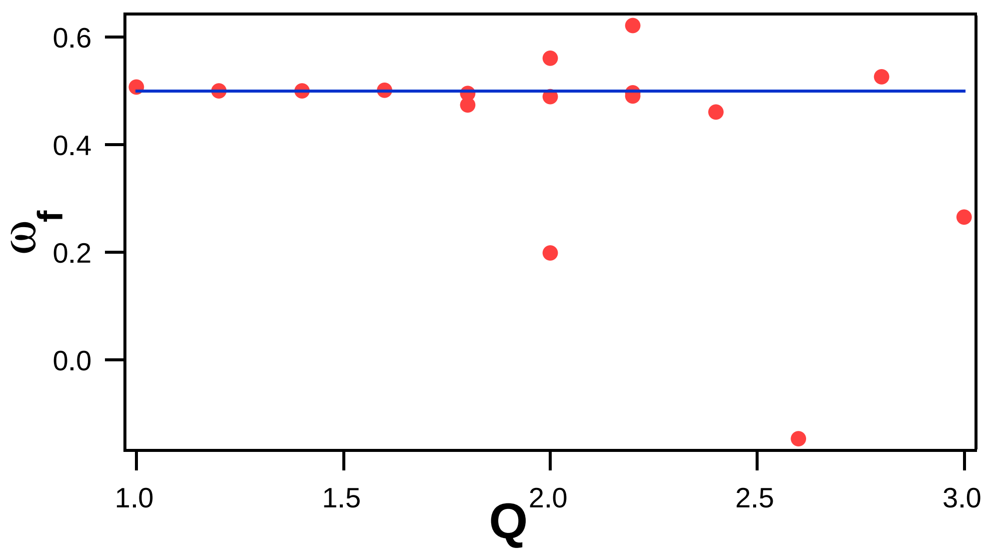
Performance in white noise uncorrected and corrected



Fitted Q vs actual Q
(no noise)



Fitted frequency vs actual Q (no noise)



Review

Possible to approximate chaotic signal with UPO's.

Degree of approximation depends on amount of computation.

Works best with dissimilar chaotic signals.

<u>Big Problem:</u>

Searching through UPO sequences is slow

Future questions

• **What is minimum SNR for self sync?**

• **Are there properties of chaotic signals that are preserved under heavy filtering?**

• **How can properties of Dynamical Systems be used to improve communications?**

• **Can chaos do anything better? Are we trying to force chaos into the wrong application?**